Completing the Square

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PRE-CALCULUS 11
QUADRATIC EQUATIONS
COMPLETING THE SQUARE

A. Definitions

1. **quadratic equation:** an equation that contains a squared variable.
2. **completing the square:** a method of solving a quadratic equation that is not easily factored.
3. **roots/zeros:** the answer(s) to a quadratic equation.

B. Solving Equations by Completing the Square when $a \neq 1$

1) $\frac{1}{2}x^2 + 6x - 1 = 0$

$+1 \quad +1$

$\frac{1}{2}x^2 + 6x = 1$

$\frac{1}{2}(x^2 + 12x) = 1$

$\frac{1}{2}(x^2 + 12x + 36) = 1 + 18$

$\frac{1}{2}(x+6)^2 = 19$ $\div \frac{1}{2}$.

$\div \frac{1}{2}$.

$\pm\sqrt{(x+6)^2} = \pm\sqrt{38}$

$x + 6 = \pm\sqrt{38}$

$-6 \quad -6$

$\boxed{x = -6 \pm \sqrt{38}}$

2) $-3x^2 - 9x - \frac{3}{4} = 0$

$+\frac{3}{4} \quad +\frac{3}{4}$

$-3x^2 - 9x = \frac{3}{4}$

$-3(x^2 + 3x) = \frac{3}{4}$

$-3(x^2 + 3x + \frac{9}{4}) = \frac{3}{4} - \frac{27}{4}$

$\frac{-3}{-3}(x + \frac{3}{2})^2 = \frac{-6}{-3}$

$\pm\sqrt{(x + \frac{3}{2})^2} = \pm\sqrt{2}$

$x + \frac{3}{2} = \pm\sqrt{2}$

$-\frac{3}{2} \quad -\frac{3}{2}$

$\boxed{x = -\frac{3}{2} \pm \sqrt{2}}$

Completing the Square Method

a) Move the "c" value to the opposite side.

b) Factor out the "a" value

c) Make the left side into a perfect square trinomial. Balance both sides.

d) Factor the perfect square trinomial.

e) Isolate the squared term and solve.

3) $-2x^2 + 7 = -3x$

$-2x^2 - 3x + 7 = 0$

$-7 \quad\quad -7$

$-2x^2 - 3x = -7$

$-2\left(x^2 + \frac{3}{2}x\right) = -7$

$-2\left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) = -7 - \frac{9}{8}$

$\frac{-2}{-2}\left(x + \frac{3}{4}\right)^2 = -\frac{65}{4} \div -2$

$\pm\sqrt{\left(x + \frac{3}{4}\right)^2} = \pm\sqrt{\frac{65}{16}}$

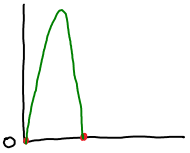
$x + \frac{3}{4} = \pm\frac{\sqrt{65}}{4}$

$\quad\quad -\frac{3}{4} \quad -\frac{3}{4}$

$x = -\frac{3}{4} \pm \frac{\sqrt{65}}{4}$

$$x = \frac{-3 \pm \sqrt{65}}{4}$$

4) A football is kicked vertically. The approximate height of the football (h *metres*), after (t *seconds*) is modeled using the formula $h = 1 + 20t - 5t^2$. When will the football hit the ground? Give your answer to the nearest tenth of a second.



$h = 1 + 20t - 5t^2$

$0 = 1 + 20t - 5t^2$

$+5t^2 - 20t \quad -1 \quad -20t \quad +5t^2$

-1

$5t^2 - 20t - 1 = 0$

$+1 \quad\quad +1$

$5t^2 - 20t = 1$

$5(t^2 - 4t) = 1$

$5(t^2 - 4t + 4) = 1 + 20$

$\frac{5}{5}(t - 2)^2 = \frac{21}{5}$

$\pm\sqrt{(t-2)^2} = \pm\sqrt{\frac{21}{5}}$

$t - 2 = \pm\sqrt{\frac{21}{5}}$

$+2 \quad\quad +2$

$t = 2 \pm \sqrt{\frac{21}{5}}$

$t = 2 + \sqrt{\frac{21}{5}}$

$t = 4.0493\ldots$

$t = 2 - \sqrt{\frac{21}{5}}$

$t = -0.0493\ldots$

Can't have negative time.

$$t = 4.0 \text{ seconds}$$

Assignment: Completing the Square Assignment #1 – 17

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A. Solve the following quadratic equations by completing the square.

1) $x^2 - 2x - 4 = 0$

2) $x^2 + 6x + 4 = 0$

3) $x^2 + 22 = 10x$

4) $x^2 - 5x - 8 = 0$

5) $2x^2 - 8x - 6 = 0$

6) $2x^2 + 12x - 2 = 0$

7) $3x^2 - 24x = 12$

8) $-x^2 - 7x - 7 = 0$

9) $-2x^2 - 10 = -20x$

10) $\frac{1}{2}x^2 - 2x - 8 = 0$

11) $\frac{1}{3}x^2 + 3x - 1 = 0$

12) $\frac{1}{2}x^2 + 6x + 3 = 0$

13) $-\frac{1}{2}x^2 - 6x + 5 = 0$

14) $-x^2 + 9x + 1 = 0$

15) $x^2 + 5x - 3 = 0$

16) $2x^2 - 10x - 20 = 4x^2 + 6x$

17) When the square of a number is added to the number, the sum is 3. What is the number?

Answers

1) $1 \pm \sqrt{5}$

2) $-3 \pm \sqrt{5}$

3) $5 \pm \sqrt{3}$

4) $\frac{5 \pm \sqrt{57}}{2}$

5) $2 \pm \sqrt{7}$

6) $-3 \pm \sqrt{10}$

7) $4 \pm 2\sqrt{5}$

8) $\frac{-7 \pm \sqrt{21}}{2}$

9) $5 \pm 2\sqrt{5}$

10) $2 \pm 2\sqrt{5}$

11) $\frac{-9 \pm \sqrt{93}}{2}$

12) $-6 \pm \sqrt{30}$

13) $-6 \pm \sqrt{46}$

14) $\frac{9 \pm \sqrt{85}}{2}$

15) $\frac{-5 \pm \sqrt{37}}{2}$

16) $-4 \pm \sqrt{6}$

17) $\frac{-1 \pm \sqrt{13}}{2}$