Dividing Radicals Part 2

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10:20 AM

PRE-CALCULUS 11
RADICALS
DIVIDING RADICALS PART 2

A. Definitions

1. **radical:** a mathematical symbol representing a root.
2. **rationalize the denominator:** removing a radical from the denominator of a fraction.

B. Dividing Complex Radicals (Binomial Denominators)

1. Rationalize the denominator.

a) $\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ $(\sqrt{6}+\sqrt{2})$ ← conjugate

$= \frac{\sqrt{12}+\sqrt{4}}{\sqrt{36}+\sqrt{12}-\sqrt{12}-\sqrt{4}}$

$= \frac{\sqrt{4}\cdot\sqrt{3}+\sqrt{4}}{\sqrt{36}+\sqrt{4}\sqrt{3}-\sqrt{4}\cdot\sqrt{3}-\sqrt{4}}$

$= \frac{2\sqrt{3}+2}{6+2\sqrt{3}-2\sqrt{3}-2}$

$= \frac{2+2\sqrt{3}}{4} \overset{\div 2}{\overset{\div 2}{}}$ $= \boxed{\frac{1+\sqrt{3}}{2}}$

b) $\frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}}$ $(\sqrt{5}-\sqrt{2})$

$= \frac{3\sqrt{15}-3\sqrt{6}}{\sqrt{25}-\sqrt{10}+\sqrt{10}-\sqrt{4}}$

$= \frac{3\sqrt{15}-3\sqrt{6}}{5-2}$

$= \frac{3\sqrt{15}-3\sqrt{6}}{3} \overset{\div 3}{\overset{\div 3}{}}$ $= \boxed{\sqrt{15}-\sqrt{6}}$

c) $\frac{3}{2\sqrt{5}-\sqrt{3}}$ $(2\sqrt{5}+\sqrt{3})$

$= \frac{6\sqrt{5}+3\sqrt{3}}{4\cdot\sqrt{25}+2\sqrt{15}-2\sqrt{15}-\sqrt{9}}$

$= \frac{6\sqrt{5}+3\sqrt{3}}{20-3}$

$= \boxed{\frac{6\sqrt{5}+3\sqrt{3}}{17}}$

To Solve

a) To remove the roots from a binomial denominator we multiply by the conjugate.

b) Simplify the fraction if possible

d) $\dfrac{(\sqrt{5}+2)(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$

$= \dfrac{\sqrt{25}+2\sqrt{5}+2\sqrt{5}+4}{\sqrt{25}+2\sqrt{5}-2\sqrt{5}-4}$

$= \dfrac{5+2\sqrt{5}+2\sqrt{5}+4}{5+2\sqrt{5}-2\sqrt{5}-4}$

$= \dfrac{9+4\sqrt{5}}{1} = \boxed{9+4\sqrt{5}}$

e) $\dfrac{(2\sqrt{6}+\sqrt{2})(6\sqrt{6}+2\sqrt{2})}{(6\sqrt{6}-2\sqrt{2})(6\sqrt{6}+2\sqrt{2})}$

$= \dfrac{12\sqrt{36}+4\sqrt{12}+6\sqrt{12}+2\sqrt{4}}{36\sqrt{36}+12\sqrt{12}-12\sqrt{12}-4\sqrt{4}}$

$= \dfrac{12\sqrt{36}+4\sqrt{4}\cdot\sqrt{3}+6\sqrt{4}\cdot\sqrt{3}+2\sqrt{4}}{36\sqrt{36}-4\sqrt{4}}$

$= \dfrac{72+8\sqrt{3}+12\sqrt{3}+4}{216-8} = \dfrac{76+20\sqrt{3}}{208} = \boxed{\dfrac{19+5\sqrt{3}}{52}}$

f) $\dfrac{4\sqrt{3}+\sqrt{8}}{\sqrt{27}-\sqrt{32}}$ $\quad \sqrt{8}=\sqrt{4}\cdot\sqrt{2}$, $\sqrt{9}\cdot\sqrt{3}$, $\sqrt{16}\cdot\sqrt{2}$

Try simplifying first

$= \dfrac{(4\sqrt{3}+2\sqrt{2})(3\sqrt{3}+4\sqrt{2})}{(3\sqrt{3}-4\sqrt{2})(3\sqrt{3}+4\sqrt{2})}$

$= \dfrac{12\sqrt{9}+16\sqrt{6}+6\sqrt{6}+8\sqrt{4}}{9\sqrt{9}+12\sqrt{6}-12\sqrt{6}-16\sqrt{4}}$

$= \dfrac{36+16\sqrt{6}+6\sqrt{6}+16}{27-32}$

$= \dfrac{52+22\sqrt{6}}{-5} = \dfrac{52+22\sqrt{6}}{-5} = \boxed{\dfrac{-52-22\sqrt{6}}{5}}$

can't have a negative denominator

Assignment: Dividing Radicals Assignment #1, 2, 3

Assignment

1. Simplify by rationalizing the denominator.

a) $\frac{4}{\sqrt{5}-1}$ b) $\frac{1}{\sqrt{6}+2}$ c) $\frac{3}{3-\sqrt{3}}$

d) $\frac{\sqrt{7}}{\sqrt{7}-2}$ e) $\frac{3}{\sqrt{2}-\sqrt{3}}$ f) $\frac{\sqrt{2}}{\sqrt{6}+\sqrt{2}}$

2. Simplify by rationalizing the denominator.

a) $\frac{2\sqrt{3}}{3\sqrt{2}+\sqrt{3}}$ b) $\frac{3\sqrt{11}}{3\sqrt{11}+10}$

c) $\frac{\sqrt{2}}{\sqrt{12}-\sqrt{8}}$ d) $\frac{\sqrt{7}}{4-\sqrt{14}}$

3. Simplify leaving an integer in the denominator.

a) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

b) $\dfrac{\sqrt{5}-2}{\sqrt{5}-1}$

c) $\dfrac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

d) $\dfrac{5-\sqrt{10}}{3+\sqrt{10}}$

e) $\dfrac{\sqrt{11}+5\sqrt{2}}{\sqrt{11}-2\sqrt{2}}$

f) $\dfrac{2\sqrt{6}-\sqrt{3}}{3\sqrt{3}+\sqrt{6}}$

g) $\dfrac{\sqrt{30}+3\sqrt{3}}{\sqrt{30}-3\sqrt{3}}$

h) $\dfrac{3\sqrt{5}-2\sqrt{3}}{3\sqrt{5}+2\sqrt{3}}$

Extension Questions.

12. Simplify by rationalizing the denominator.

a) $\frac{3}{2\sqrt{x}+3}$ b) $\frac{x+\sqrt{10}}{x-\sqrt{10}}$ c) $\frac{\sqrt{k}+\sqrt{2}}{\sqrt{k}-\sqrt{2}}$

Multiple Choice **13.** $\frac{p}{q-\sqrt{r}}$, expressed with a rational denominator, may be written as

A. $\frac{p}{q^2-r}$ B. $\frac{p(q+\sqrt{r})}{q^2-r^2}$

C. $\frac{p(q+\sqrt{r})}{q^2-r}$ D. $\frac{p(q-\sqrt{r})}{q^2+r}$

Answer Key

1. a) $\sqrt{5}+1$ b) $\frac{\sqrt{6}-2}{2}$ c) $\frac{3+\sqrt{3}}{2}$ d) $\frac{7+2\sqrt{7}}{3}$ e) $-3\sqrt{2}-3\sqrt{3}$ f) $\frac{\sqrt{3}-1}{2}$

2. a) $\frac{2\sqrt{6}-2}{5}$ b) $30\sqrt{11}-99$ c) $\frac{\sqrt{6}+2}{2}$ d) $\frac{4\sqrt{7}+7\sqrt{2}}{2}$

3. a) $2-\sqrt{3}$ b) $\frac{3-\sqrt{5}}{4}$ c) $2+\sqrt{3}$ d) $8\sqrt{10}-25$ e) $\frac{31+7\sqrt{22}}{3}$

f) $\sqrt{2}-1$ g) $19+6\sqrt{10}$ h) $\frac{19-4\sqrt{15}}{11}$

4. a) $\frac{11\sqrt{10}-30}{31}$ b) $\frac{-8-2\sqrt{6}}{5}$ **5.** i) $\frac{15-5\sqrt{3}}{6}$ m. ii) 1.06 m.

6. $\frac{32-10\sqrt{10}}{3}$ units. **7.** C **8.** A **9.** D

10.

2	.	0	

11.

1	.	2	0

12.a) $\frac{6\sqrt{x}-9}{4x-9}$ b) $\frac{x^2+2x\sqrt{10}+10}{x^2-10}$ c) $\frac{k+2\sqrt{2k}+2}{k-2}$ **13.** C