Interpreting the Discriminant

November-21-18
8:32 AM

PRE-CALCULUS 11
QUADRATIC EQUATIONS
INTERPRETING THE DISCRIMINANT

A. Definitions

1. **rational number:** a number that can be written in the form of a fraction.
2. **irrational number:** a number that cannot be written in the form of a fraction.

B. Understanding the Nature of Quadratic Roots

Sometimes when dealing with an equation, we would like to answer the question "How many solutions (roots) does the equation have?" versus "What are the solutions (roots) to the equation?" The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can be used to answer the second question. In the quadratic formula the part under the root $b^2 - 4ac$ is called the **discriminant**. The sign of the discriminant can be used to determine the nature of the roots of the equation.

Sign of discriminant	Nature of the Roots
$b^2 - 4ac < 0$ (negative)	No solutions (No roots)
$b^2 - 4ac = 0$	One Solution (One root)
$b^2 - 4ac > 0$ (positive)	Two Solutions (Two roots)

C. Examples

1. Determine the value of the discriminant and whether the following has one, two or no real roots.

a) $4x^2 - 12x + 9 = 0$
$a = 4, b = -12, c = 9$

$b^2 - 4ac$
$(-12)^2 - 4(4)(9)$
$= \boxed{0}$ Value of the discriminant

One Root.

b) $x^2 - 6x + 4 = 0$
$a = 1, b = -6, c = 4$

$b^2 - 4ac$
$(-6)^2 - 4(1)(4)$
$= \boxed{20}$

Two Roots

c) $5x^2 - 8x + 6 = 0$
$a = 5, b = -8, c = 6$

$b^2 - 4ac$
$(-8)^2 - 4(5)(6)$
$= \boxed{-56}$

No Real Roots

2. Determine if the roots are rational or irrational and can be solved by factoring.

a) $3x^2 - 8x - 1 = 0$

$a = 3, b = -8, c = -1$

$b^2 - 4ac.$

$(-8)^2 - 4(3)(-1)$

$= 76$

$= \sqrt{76}$

Irrational Number

Will not factor.

b) $5x^2 - 13x + 6 = 0$

$a = 5, b = -13, c = 6$

$b^2 - 4ac.$

$(-13)^2 - 4(5)(6).$

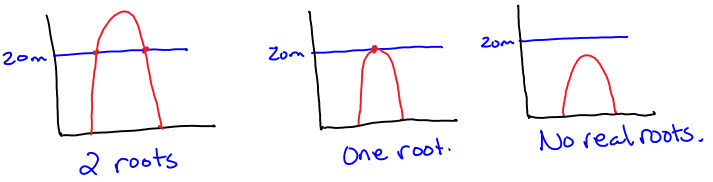
$= 49$

$= \sqrt{49}$

Rational Number

This is factorable.

3. When a ball is kicked vertically into the air, its height, h metres, is given by the formula $h = 12t - 4.9t^2$, where t is the time in seconds. Will the ball reach a height of 20 metres?



2 roots

One root.

No real roots.

$h = 12t - 4.9t^2$

$20 = 12t - 4.9t^2.$

$-12t \quad -12t \quad +4.9t^2 \quad +4.9t^2.$

$4.9t^2 - 12t + 20 = 0.$

$a = 4.9, b = -12, c = 20.$

$b^2 - 4ac.$

$(-12)^2 - 4(4.9)(20).$

$= -248.$

It will not reach 20 m.

Assignment: Pg. 232 #4, 5, 6, 11, 12, 13