Lesson 2.2 Exercises, pages 100–105

A

3. Write each mixed radical as an entire radical.

a) $6\sqrt{5} = \sqrt{36} \cdot \sqrt{5}$
$= \sqrt{180}$

b) $3\sqrt[3]{4} = \sqrt[3]{3^3} \cdot \sqrt[3]{4}$
$= \sqrt[3]{27} \cdot \sqrt[3]{4}$
$= \sqrt[3]{108}$

c) $-2\sqrt[3]{5} = \sqrt[3]{(-2)^3} \cdot \sqrt[3]{5}$
$= \sqrt[3]{-8} \cdot \sqrt[3]{5}$
$= \sqrt[3]{-40}$

d) $5\sqrt[4]{2} = \sqrt[4]{5^4} \cdot \sqrt[4]{2}$
$= \sqrt[4]{625} \cdot \sqrt[4]{2}$
$= \sqrt[4]{1250}$

4. Write each entire radical as a mixed radical, if possible.

a) $\sqrt{54} = \sqrt{9 \cdot 6}$
$= 3\sqrt{6}$

b) $\sqrt[3]{96} = \sqrt[3]{8 \cdot 12}$
$= 2\sqrt[3]{12}$

c) $\sqrt[3]{-81} = \sqrt[3]{-27 \cdot 3}$
$= -3\sqrt[3]{3}$

d) $\sqrt[4]{47}$ $\sqrt[4]{47}$ cannot be written as a mixed radical because 47 does not have any factors that are perfect fourth powers

5. Arrange in order from least to greatest.

a) $6\sqrt[3]{7}, 11\sqrt[3]{7}, 5\sqrt[3]{7}$

Each mixed radical is a multiple of $\sqrt[3]{7}$, so compare the coefficients: $5 < 6 < 11$
So, from least to greatest:
$5\sqrt[3]{7}, 6\sqrt[3]{7}, 11\sqrt[3]{7}$

b) $\sqrt{28}, 2\sqrt{5}, 3\sqrt{3}$

Each radical has index 2.
Write each mixed radical as an entire radical.

$\sqrt{28}$

$$\begin{aligned} 2\sqrt{5} &= \sqrt{2^2}\cdot\sqrt{5} \\ &= \sqrt{4\cdot 5} \\ &= \sqrt{20} \end{aligned} \qquad \begin{aligned} 3\sqrt{3} &= \sqrt{3^2}\cdot\sqrt{3} \\ &= \sqrt{9\cdot 3} \\ &= \sqrt{27} \end{aligned}$$

Compare the radicands: $20 < 27 < 28$
So, from least to greatest: $2\sqrt{5}, 3\sqrt{3}, \sqrt{28}$

B

6. For which values of the variable is each radical defined?
Justify your answers.

a) $\sqrt{18x^2}$

$\sqrt{18x^2} \in \mathbb{R}$ when $18x^2 \geq 0$.
$18 > 0$ and $x^2 \geq 0$
So, $\sqrt{18x^2}$ is defined for $x \in \mathbb{R}$.

b) $\sqrt{-2a^3}$

$\sqrt{-2a^3} \in \mathbb{R}$ when $-2a^3 \geq 0$.
$-2 < 0$, so $a^3 \leq 0$; that is, $a \leq 0$
So, $\sqrt{-2a^3}$ is defined for $a \leq 0$.

c) $\sqrt[3]{4c^3}$

Since the cube root of a number is defined for all real numbers,
$\sqrt[3]{4c^3} \in \mathbb{R}$ when $4c^3 \in \mathbb{R}$.
So, $\sqrt[3]{4c^3}$ is defined for $c \in \mathbb{R}$.

d) $\sqrt[4]{18z^5}$

$\sqrt[4]{18z^5} \in \mathbb{R}$ when $18z^5 \geq 0$.
$18 > 0$, so $z^5 \geq 0$; that is, $z \geq 0$
So, $\sqrt[4]{18z^5}$ is defined for $z \geq 0$.

7. Write each mixed radical as an entire radical.

a) $3\sqrt{\frac{7}{3}}$

$$\begin{aligned} &= \sqrt{3^2}\cdot\sqrt{\frac{7}{3}} \\ &= \sqrt{9\left(\frac{7}{3}\right)} \\ &= \sqrt{21} \end{aligned}$$

b) $-\frac{2}{3}\sqrt{\frac{5}{8}}$

$$\begin{aligned} &= -\sqrt{\left(\frac{2}{3}\right)^2}\cdot\sqrt{\frac{5}{8}} \\ &= -\sqrt{\left(\frac{4}{9}\right)\left(\frac{5}{8}\right)} \\ &= -\sqrt{\frac{5}{18}} \end{aligned}$$

c) $-\frac{2}{3}\sqrt[3]{\frac{2}{3}}$

$$\begin{aligned} &= \sqrt[3]{\left(-\frac{2}{3}\right)^3}\cdot\sqrt[3]{\frac{2}{3}} \\ &= \sqrt[3]{\left(-\frac{8}{27}\right)\left(\frac{2}{3}\right)} \\ &= \sqrt[3]{-\frac{16}{81}} \end{aligned}$$

8. Write each entire radical as a mixed radical, if possible.

a) $\sqrt{\frac{125}{4}}$

$$\begin{aligned} &= \frac{\sqrt{25\cdot 5}}{\sqrt{4}} \\ &= \frac{5\sqrt{5}}{2} \end{aligned}$$

b) $\sqrt[4]{\frac{32}{243}}$

$$\begin{aligned} &= \frac{\sqrt[4]{16\cdot 2}}{\sqrt[4]{81\cdot 3}} \\ &= \frac{2\sqrt[4]{2}}{3\sqrt[4]{3}} \\ &= \frac{2}{3}\sqrt[4]{\frac{2}{3}} \end{aligned}$$

c) $\sqrt[3]{-\frac{64}{81}}$

$$\begin{aligned} &= \frac{\sqrt[3]{-64}}{\sqrt[3]{27\cdot 3}} \\ &= \frac{-4}{3\sqrt[3]{3}} \end{aligned}$$

9. Arrange in order from greatest to least.

a) $3\sqrt[3]{5}, 2\sqrt{11}, 4\sqrt[4]{8}$

The mixed radicals have different indices, so use a calculator.

$3\sqrt[3]{5} = 5.1299\ldots$

$2\sqrt{11} = 6.6332\ldots$

$4\sqrt[4]{8} = 6.7271\ldots$

Compare the decimals: $6.7271 > 6.6332 > 5.1299$

So, from greatest to least: $4\sqrt[4]{8}, 2\sqrt{11}, 3\sqrt[3]{5}$

b) $4\sqrt{6}, 4\sqrt{7}, 4\sqrt{2}, 4\sqrt{5}$

Each mixed radical has index 2 and coefficient 4.

So, compare the radicands: $7 > 6 > 5 > 2$

So, from greatest to least: $4\sqrt{7}, 4\sqrt{6}, 4\sqrt{5}, 4\sqrt{2}$

c) $3\sqrt[3]{5}, 2\sqrt[3]{12}, 4\sqrt[3]{2}, 3\sqrt[3]{4}$

Each radical has index 3.

Write each mixed radical as an entire radical.

$$3\sqrt[3]{5} = \sqrt[3]{3^3 \cdot 5} = \sqrt[3]{135} \qquad 2\sqrt[3]{12} = \sqrt[3]{2^3 \cdot 12} = \sqrt[3]{96}$$

$$4\sqrt[3]{2} = \sqrt[3]{4^3 \cdot 2} = \sqrt[3]{128} \qquad 3\sqrt[3]{4} = \sqrt[3]{3^3 \cdot 4} = \sqrt[3]{108}$$

Compare the radicands: $135 > 128 > 108 > 96$

So, from greatest to least: $3\sqrt[3]{5}, 4\sqrt[3]{2}, 3\sqrt[3]{4}, 2\sqrt[3]{12}$

10. Write the values of the variable for which each radical is defined. Simplify the radical, if possible.

a) $\sqrt[3]{-8x^2}$

Since the cube root of a number is defined for all real values of x, the radical is defined for $x \in \mathbb{R}$.

$$\sqrt[3]{-8x^2} = \sqrt[3]{-8 \cdot x^2} = -2\sqrt[3]{x^2}$$

b) $\sqrt{48b^4}$

$\sqrt{48b^4} \in \mathbb{R}$ when $48b^4 \geq 0$.

$48 > 0$ and $b^4 \geq 0$

So, $\sqrt{48b^4}$ is defined for $b \in \mathbb{R}$.

$$\sqrt{48b^4} = \sqrt{16 \cdot 3 \cdot b^4} = 4b^2\sqrt{3}$$

c) $\sqrt[4]{16r^4}$

$\sqrt[4]{16r^4} \in \mathbb{R}$ when $16r^4 \geq 0$.

$16 > 0$ and $r^4 \geq 0$

So, $\sqrt[4]{16r^4}$ is defined for $r \in \mathbb{R}$.

$$\sqrt[4]{16r^4} = \sqrt[4]{16 \cdot r^4} = 2|r|$$

d) $\sqrt[4]{125x^3}$

$\sqrt[4]{125x^3} \in \mathbb{R}$ when $125x^3 \geq 0$.

$125 > 0$ so $x^3 \geq 0$; that is, $x \geq 0$

So, $\sqrt[4]{125x^3}$ is defined for $x \geq 0$.

$\sqrt[4]{125x^3}$ cannot be simplified because the radicand does not have any factors that are perfect fourth powers.

11. For which values of the variable is each radical defined? Rewrite the radical in simplest form.

a) $\sqrt[3]{\frac{27x^2}{16}}$

Since the cube root of a number is defined for all real values of x, the radical is defined for $x \in \mathbb{R}$.

$\sqrt[3]{\frac{27x^2}{16}} = \sqrt[3]{\frac{27 \cdot x^2}{8 \cdot 2}}$

$= \frac{3}{2}\sqrt[3]{\frac{x^2}{2}}$

b) $\sqrt[3]{\frac{-48c^5}{125}}$

Since the cube root of a number is defined for all real values of c, the radical is defined for $c \in \mathbb{R}$.

$\sqrt[3]{\frac{-48c^5}{125}} = \sqrt[3]{\frac{-8c^3 \cdot 6c^2}{125}}$

$= -\frac{2}{5}c\sqrt[3]{6c^2}$

c) $\sqrt[4]{\frac{81y^5}{16}}$

$\sqrt[4]{\frac{81y^5}{16}} \in \mathbb{R}$ when $y^5 \geq 0$; that is, $y \geq 0$

$\sqrt[4]{\frac{81y^5}{16}} = \sqrt[4]{\frac{81y^4 \cdot y}{16}}$

$= \frac{3}{2}y\sqrt[4]{y}$

12. Determine whether each statement is:

- always true
- sometimes true
- never true

Justify your answer.

a) $(-x)^2 = x^2$ Always true; a number multiplied by itself is always positive.

b) $\sqrt{x^2} = \pm x$

Sometimes true; $\sqrt{x^2} = |x|$, and $|x| \geq 0$, so when $x \neq 0$, $\sqrt{x^2} \neq \pm x$.

But, when $x = 0$, $\sqrt{x^2} = \pm x$

13. In the far north, ice roads are often the only way to get supplies to mines. The formula $t = 4\sqrt{m}$ is used to determine the minimum thickness of ice, t inches, needed for an ice road to support a mass of m tons.

a) What is the minimum thickness of ice needed for a mass of 10 T?

Use the formula $t = 4\sqrt{m}$. Substitute: $m = 10$

$t = 4\sqrt{10}$

So, the minimum thickness of ice needed is $4\sqrt{10}$ in.

b) What is the maximum mass for ice that is 10 in. thick?

Use the formula $t = 4\sqrt{m}$. Substitute: $t = 10$

$10 = 4\sqrt{m}$

$\frac{5}{2} = \sqrt{m}$

$\left(\frac{5}{2}\right)^2 = (\sqrt{m})^2$

$m = \frac{25}{4}$, or 6.25

So, the maximum mass is 6.25 T.

c) Suppose the mass of a load is multiplied by 4. How will the thickness of the ice have to increase? Explain your thinking.

When the mass is multiplied by 4, $t = 4\sqrt{4m}$

$= 4 \cdot 2\sqrt{m}$

Compare this to the original equation: $t = 4\sqrt{m}$

The thickness of the ice will be multiplied by 2, which is $\sqrt{4}$.

C

14. For which values of the variables is each radical defined? Simplify the radical, if possible.

a) $\sqrt{98a^3b^6}$

$\sqrt{98a^3b^6} \in \mathbb{R}$ when $98a^3b^6 \geq 0$.

$98 > 0$ and $b^6 \geq 0$

So, $a^3 \geq 0$; that is, $a \geq 0$

$\sqrt{98a^3b^6}$ is defined for $a \geq 0$, $b \in \mathbb{R}$.

$\sqrt{98a^3b^6} = \sqrt{49 \cdot 2 \cdot a^2b^6 \cdot a}$

$= 7a|b^3|\sqrt{2a}$

b) $\sqrt[3]{-40x^3y^5}$

Since the cube root of a number is defined for all real numbers,

$\sqrt[3]{-40x^3y^5}$ is defined for $x \in \mathbb{R}$, $y \in \mathbb{R}$.

$\sqrt[3]{-40x^3y^5} = \sqrt[3]{-8 \cdot 5 \cdot x^3y^3 \cdot y^2}$

$= -2xy\sqrt[3]{5y^2}$

c) $\sqrt[4]{48r^4s^8}$

$\sqrt[4]{48r^4s^8} \in \mathbb{R}$ when $48r^4s^8 \geq 0$.

$48 > 0$, $r^4 \geq 0$, and $s^8 \geq 0$

So, $\sqrt[4]{48r^4s^8}$ is defined for $r \in \mathbb{R}$, $s \in \mathbb{R}$.

$\sqrt[4]{48r^4s^8} = \sqrt[4]{16 \cdot 3 \cdot r^4s^8}$

$= 2|r|s^2\sqrt[4]{3}$

d) $\sqrt[3]{128m^2n^4}$

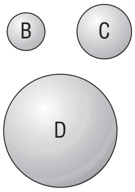
Since the cube root of a number is defined for all real numbers,

$\sqrt[3]{128m^2n^4}$ is defined for $m \in \mathbb{R}$, $n \in \mathbb{R}$.

$\sqrt[3]{128m^2n^4} = \sqrt[3]{64 \cdot 2 \cdot n^3 \cdot m^2n}$

$= 4n\sqrt[3]{2m^2n}$

15. The surface area of sphere B is A square units. The surface area of sphere C is twice that of sphere B. The surface area of sphere D is 9 times that of sphere B. Determine the radii of spheres C and D in terms of A.



Formula for surface area of a sphere: $SA = 4\pi r^2$
Sphere B has surface area A square units.

Sphere C
Surface area: $2A$
So, $2A = 4\pi r^2$

$$\frac{2A}{4\pi} = r^2$$
$$\frac{A}{2\pi} = r^2$$
$$\sqrt{\frac{A}{2\pi}} = r$$

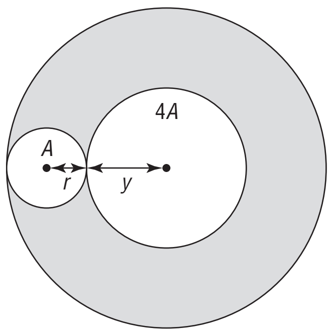
Sphere C has radius $\sqrt{\frac{A}{2\pi}}$ units.

Sphere D
Surface area: $9A$
So, $9A = 4\pi r^2$

$$\frac{9A}{4\pi} = r^2$$
$$\sqrt{\frac{9A}{4\pi}} = r$$
$$\frac{3}{2}\sqrt{\frac{A}{\pi}} = r$$

Sphere D has radius $\frac{3}{2}\sqrt{\frac{A}{\pi}}$ units.

16. A circle with radius r and area A touches another circle with radius y and area $4A$. A larger circle is drawn around the other two as shown.



a) Determine an expression for the value of y in terms of r. State the restrictions on the variables.

Formula for area of a circle: $A = \pi r^2$
Since r and y are lengths, $r > 0$ and $y > 0$.
Area of small circle: $A = \pi r^2$
Area of middle circle: $4A = \pi y^2$
To determine y in terms of r, substitute $A = \pi r^2$ in $4A = \pi y^2$, then solve for y.

$$4A = \pi y^2$$
$$4(\pi r^2) = \pi y^2$$
$$\frac{4\pi r^2}{\pi} = y^2$$
$$4r^2 = y^2$$
$$y = \sqrt{4r^2}$$
$$y = 2r$$

b) Determine the area of the shaded region in terms of A.

Area of shaded region
$= $ Area of large circle $-$ area of 2 smaller circles
$= \pi(y + 2r)^2 - 4A - A$ Substitute: $y = 2r$
$= \pi(2r + 2r)^2 - 5A$
$= \pi(4r)^2 - 5A$
$= 16\pi r^2 - 5A$ Substitute: $\pi r^2 = A$
$= 16A - 5A$
$= 11A$
The area of the shaded region is $11A$.

