Lesson 2.4 Exercises, pages 126–133

A

3. Expand and simplify.

a) $\sqrt{6}(\sqrt{5} + 2)$

$= \sqrt{6}(\sqrt{5}) + \sqrt{6}(2)$
$= \sqrt{30} + 2\sqrt{6}$

b) $\sqrt{5}(\sqrt{2} - 4)$

$= \sqrt{5}(\sqrt{2}) - \sqrt{5}(4)$
$= \sqrt{10} - 4\sqrt{5}$

c) $\sqrt{2}(-7 + \sqrt{2})$

$= \sqrt{2}(-7) + \sqrt{2}(\sqrt{2})$
$= -7\sqrt{2} + 2$

d) $-\sqrt{3}(3 + \sqrt{8})$

$= -\sqrt{3}(3) - \sqrt{3}(\sqrt{8})$
$= -3\sqrt{3} - \sqrt{24}$
$= -3\sqrt{3} - 2\sqrt{6}$

4. Expand and simplify.

a) $\sqrt{6}(\sqrt{3} + \sqrt{2})$

$= \sqrt{6}(\sqrt{3}) + \sqrt{6}(\sqrt{2})$
$= \sqrt{18} + \sqrt{12}$
$= 3\sqrt{2} + 2\sqrt{3}$

b) $-2\sqrt{7}(3\sqrt{2} - \sqrt{7})$

$= -2\sqrt{7}(3\sqrt{2}) - 2\sqrt{7}(-\sqrt{7})$
$= -6\sqrt{14} + 2(7)$
$= -6\sqrt{14} + 14$

5. Expand and simplify.

a) $(\sqrt{3} + 2)(\sqrt{3} - 2)$

$= \sqrt{3}(\sqrt{3} - 2) + 2(\sqrt{3} - 2)$
$= \sqrt{3}(\sqrt{3}) - \sqrt{3}(2) + 2(\sqrt{3}) - 2(2)$
$= 3 - 2\sqrt{3} + 2\sqrt{3} - 4$
$= -1$

b) $(\sqrt{5} + 2)^2$

$= (\sqrt{5} + 2)(\sqrt{5} + 2)$
$= \sqrt{5}(\sqrt{5} + 2) + 2(\sqrt{5} + 2)$
$= \sqrt{5}(\sqrt{5}) + \sqrt{5}(2) + 2(\sqrt{5}) + 2(2)$
$= 5 + 2\sqrt{5} + 2\sqrt{5} + 4$
$= 9 + 4\sqrt{5}$

c) $(\sqrt{3} + \sqrt{2})^2$

$= (\sqrt{3} + \sqrt{2})(\sqrt{3} + \sqrt{2})$
$= \sqrt{3}(\sqrt{3} + \sqrt{2}) + \sqrt{2}(\sqrt{3} + \sqrt{2})$
$= \sqrt{3}(\sqrt{3}) + \sqrt{3}(\sqrt{2}) + \sqrt{2}(\sqrt{3}) + \sqrt{2}(\sqrt{2})$
$= 3 + \sqrt{6} + \sqrt{6} + 2$
$= 5 + 2\sqrt{6}$

d) $(2\sqrt{3} + 3\sqrt{5})(\sqrt{3} - 2\sqrt{5})$

$= 2\sqrt{3}(\sqrt{3} - 2\sqrt{5}) + 3\sqrt{5}(\sqrt{3} - 2\sqrt{5})$
$= 2(3) - 4(\sqrt{15}) + 3\sqrt{15} - 6(5)$
$= 6 - 4\sqrt{15} + 3\sqrt{15} - 30$
$= -24 - \sqrt{15}$

6. Rationalize the denominator.

a) $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{2\sqrt{3}}{3}$

b) $\frac{1}{5\sqrt{3}} = \frac{1}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{\sqrt{3}}{5(3)}$
$= \frac{\sqrt{3}}{15}$

B

7. Expand and simplify.

a) $(\sqrt{3} + 8)(2\sqrt{3} - 1) - \sqrt{3}(7\sqrt{3} - 4)$

$= \sqrt{3}(2\sqrt{3} - 1) + 8(2\sqrt{3} - 1) - \sqrt{3}(7\sqrt{3}) - \sqrt{3}(-4)$
$= 2(3) - \sqrt{3} + 16\sqrt{3} - 8 - 7(3) + 4\sqrt{3}$
$= 6 - \sqrt{3} + 16\sqrt{3} - 8 - 21 + 4\sqrt{3}$
$= -23 + 19\sqrt{3}$

b) $-(3\sqrt{2} - \sqrt{5})(\sqrt{2} + 7) - (2\sqrt{2} - \sqrt{5})^2$

$= (-3\sqrt{2} + \sqrt{5})(\sqrt{2} + 7) - (2\sqrt{2} - \sqrt{5})(2\sqrt{2} - \sqrt{5})$
$= -3\sqrt{2}(\sqrt{2} + 7) + \sqrt{5}(\sqrt{2} + 7) - [2\sqrt{2}(2\sqrt{2} - \sqrt{5}) - \sqrt{5}(2\sqrt{2} - \sqrt{5})]$
$= -3(2) - 21\sqrt{2} + \sqrt{10} + 7\sqrt{5} - [4(2) - 2\sqrt{10} - 2\sqrt{10} + 5]$
$= -6 - 21\sqrt{2} + \sqrt{10} + 7\sqrt{5} - 8 + 4\sqrt{10} - 5$
$= -19 - 21\sqrt{2} + 5\sqrt{10} + 7\sqrt{5}$

c) $(6\sqrt{7} - 5\sqrt{5})(6\sqrt{7} + 5\sqrt{5}) + (3\sqrt{7} + 4\sqrt{5})^2$

$= 6\sqrt{7}(6\sqrt{7} + 5\sqrt{5}) - 5\sqrt{5}(6\sqrt{7} + 5\sqrt{5}) + (3\sqrt{7} + 4\sqrt{5})(3\sqrt{7} + 4\sqrt{5})$
$= 6\sqrt{7}(6\sqrt{7}) + 6\sqrt{7}(5\sqrt{5}) - 5\sqrt{5}(6\sqrt{7}) - 5\sqrt{5}(5\sqrt{5})$
$+ 3\sqrt{7}(3\sqrt{7} + 4\sqrt{5}) + 4\sqrt{5}(3\sqrt{7} + 4\sqrt{5})$
$= 36(7) + 30\sqrt{35} - 30\sqrt{35} - 25(5) + 3\sqrt{7}(3\sqrt{7})$
$+ 3\sqrt{7}(4\sqrt{5}) + 4\sqrt{5}(3\sqrt{7}) + 4\sqrt{5}(4\sqrt{5})$
$= 252 - 125 + 9(7) + 12\sqrt{35} + 12\sqrt{35} + 16(5)$
$= 252 - 125 + 63 + 24\sqrt{35} + 80$
$= 270 + 24\sqrt{35}$

8. Identify the values of the variable for which each expression is defined, then expand and simplify.

a) $\sqrt{w}(2\sqrt{w} + 3)$

The radicands cannot be negative, so $w \geq 0$.

$\sqrt{w}(2\sqrt{w} + 3) = \sqrt{w}(2\sqrt{w}) + \sqrt{w}(3)$
$= 2w + 3\sqrt{w}$

b) $(3\sqrt{x} - 2)(2\sqrt{x} + 5)$

The radicands cannot be negative, so $x \geq 0$.

$(3\sqrt{x} - 2)(2\sqrt{x} + 5) = 3\sqrt{x}(2\sqrt{x} + 5) - 2(2\sqrt{x} + 5)$
$= 3\sqrt{x}(2\sqrt{x}) + 3\sqrt{x}(5) - 2(2\sqrt{x}) - 2(5)$
$= 6x + 15\sqrt{x} - 4\sqrt{x} - 10$
$= 6x + 11\sqrt{x} - 10$

c) $(\sqrt{c} + \sqrt{d})^2$

The radicands cannot be negative, so $c \geq 0$ and $d \geq 0$.

$(\sqrt{c} + \sqrt{d})^2 = (\sqrt{c} + \sqrt{d})(\sqrt{c} + \sqrt{d})$
$= \sqrt{c}(\sqrt{c} + \sqrt{d}) + \sqrt{d}(\sqrt{c} + \sqrt{d})$
$= c + \sqrt{cd} + \sqrt{cd} + d$
$= c + 2\sqrt{cd} + d$

d) $(2\sqrt{x} - 3\sqrt{y})(2\sqrt{x} + 3\sqrt{y})$

The radicands cannot be negative, so $x \geq 0$ and $y \geq 0$.
The expression is the binomial factors of a difference of squares.

$(2\sqrt{x} - 3\sqrt{y})(2\sqrt{x} + 3\sqrt{y}) = (2\sqrt{x})^2 - (3\sqrt{y})^2$
$= 4x - 9y$

e) $(2\sqrt{a} - \sqrt{b})(3\sqrt{a} - 4\sqrt{b}) - (\sqrt{a} - 3\sqrt{b})^2$

The radicands cannot be negative, so $a \geq 0$ and $b \geq 0$.

$$\begin{aligned}
&(2\sqrt{a} - \sqrt{b})(3\sqrt{a} - 4\sqrt{b}) - (\sqrt{a} - 3\sqrt{b})^2 \\
&= 2\sqrt{a}(3\sqrt{a} - 4\sqrt{b}) - \sqrt{b}(3\sqrt{a} - 4\sqrt{b}) - (\sqrt{a} - 3\sqrt{b})(\sqrt{a} - 3\sqrt{b}) \\
&= 2\sqrt{a}(3\sqrt{a}) - 2\sqrt{a}(4\sqrt{b}) - \sqrt{b}(3\sqrt{a}) - \sqrt{b}(-4\sqrt{b}) \\
&\quad - \left[\sqrt{a}(\sqrt{a} - 3\sqrt{b}) - 3\sqrt{b}(\sqrt{a} - 3\sqrt{b})\right] \\
&= 6a - 8\sqrt{ab} - 3\sqrt{ab} + 4b - \left[\sqrt{a}(\sqrt{a}) - \sqrt{a}(3\sqrt{b})\right. \\
&\quad \left.- 3\sqrt{b}(\sqrt{a}) - 3\sqrt{b}(-3\sqrt{b})\right] \\
&= 6a - 11\sqrt{ab} + 4b - [a - 3\sqrt{ab} - 3\sqrt{ab} + 9b] \\
&= 6a - 11\sqrt{ab} + 4b - a + 3\sqrt{ab} + 3\sqrt{ab} - 9b \\
&= 5a - 5\sqrt{ab} - 5b
\end{aligned}$$

9. Simplify. Describe the strategy you used in part c.

a) $\dfrac{2\sqrt{5} + 4}{\sqrt{5}}$

$$\begin{aligned}
&= \frac{(2\sqrt{5} + 4)}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\
&= \frac{2\sqrt{5} \cdot \sqrt{5} + 4 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} \\
&= \frac{10 + 4\sqrt{5}}{5}
\end{aligned}$$

b) $\dfrac{5\sqrt{8} - 2\sqrt{5}}{\sqrt{6}}$

$$\begin{aligned}
&= \frac{(5\sqrt{8} - 2\sqrt{5})}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} \\
&= \frac{5\sqrt{8} \cdot \sqrt{6} - 2\sqrt{5} \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} \\
&= \frac{5\sqrt{48} - 2\sqrt{30}}{6} \\
&= \frac{20\sqrt{3} - 2\sqrt{30}}{6} \\
&= \frac{10\sqrt{3} - \sqrt{30}}{3}
\end{aligned}$$

c) $\dfrac{-3\sqrt{12} + 2\sqrt{3}}{\sqrt{18}}$

$$\begin{aligned}
&= \frac{-6\sqrt{3} + 2\sqrt{3}}{3\sqrt{2}} \\
&= \frac{-4\sqrt{3}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\
&= \frac{-4\sqrt{3} \cdot \sqrt{2}}{3\sqrt{2} \cdot \sqrt{2}} \\
&= \frac{-4\sqrt{6}}{6} \\
&= \frac{-2\sqrt{6}}{3}
\end{aligned}$$

d) $\dfrac{4\sqrt{2} - 6\sqrt{5}}{2\sqrt{3}}$

$$\begin{aligned}
&= \frac{4\sqrt{2} - 6\sqrt{5}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\
&= \frac{4\sqrt{2} \cdot \sqrt{3} - 6\sqrt{5} \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} \\
&= \frac{4\sqrt{6} - 6\sqrt{15}}{6} \\
&= \frac{2\sqrt{6} - 3\sqrt{15}}{3}
\end{aligned}$$

In part c, I first simplified the radicals, then collected like terms.
I was left with an expression with a monomial in the numerator and in the denominator. I rationalized the denominator, then removed a common factor.

10. Simplify.

a) $\dfrac{-\sqrt{5}}{\sqrt{7}-3}$

$= \dfrac{-\sqrt{5}}{(\sqrt{7}-3)} \cdot \dfrac{(\sqrt{7}+3)}{(\sqrt{7}+3)}$

$= \dfrac{-\sqrt{5}(\sqrt{7}) - \sqrt{5}(3)}{(\sqrt{7})^2 - (3)^2}$

$= \dfrac{-\sqrt{35} - 3\sqrt{5}}{7-9}$

$= \dfrac{-(\sqrt{35} + 3\sqrt{5})}{-2}$

$= \dfrac{\sqrt{35} + 3\sqrt{5}}{2}$

b) $\dfrac{5\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \dfrac{(5\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \cdot \dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$

$= \dfrac{5\sqrt{3}(\sqrt{3}+\sqrt{2}) + \sqrt{2}(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2}$

$= \dfrac{5\sqrt{3}(\sqrt{3}) + 5\sqrt{3}(\sqrt{2}) + \sqrt{2}(\sqrt{3}) + \sqrt{2}(\sqrt{2})}{3-2}$

$= 15 + 5\sqrt{6} + \sqrt{6} + 2$

$= 17 + 6\sqrt{6}$

c) $\dfrac{5\sqrt{8} - 3\sqrt{2}}{\sqrt{32} + 3\sqrt{2}}$

Simplify first.

$\dfrac{5\sqrt{8} - 3\sqrt{2}}{\sqrt{32} + 3\sqrt{2}} = \dfrac{10\sqrt{2} - 3\sqrt{2}}{4\sqrt{2} + 3\sqrt{2}}$

$= \dfrac{7\sqrt{2}}{7\sqrt{2}}$

$= 1$

d) $\dfrac{6\sqrt{3} - 2}{5 + 4\sqrt{2}}$

$= \dfrac{(6\sqrt{3} - 2)}{(5 + 4\sqrt{2})} \cdot \dfrac{(5 - 4\sqrt{2})}{(5 - 4\sqrt{2})}$

$= \dfrac{6\sqrt{3}(5 - 4\sqrt{2}) - 2(5 - 4\sqrt{2})}{(5)^2 - (4\sqrt{2})^2}$

$= \dfrac{6\sqrt{3}(5) - 6\sqrt{3}(4\sqrt{2}) - 2(5) - 2(-4\sqrt{2})}{25 - 32}$

$= \dfrac{30\sqrt{3} - 24\sqrt{6} - 10 + 8\sqrt{2}}{-7}$

$= \dfrac{-30\sqrt{3} + 24\sqrt{6} + 10 - 8\sqrt{2}}{7}$

11. Rationalize each denominator. Describe any patterns in each list of quotients. Predict the next three quotients in each pattern.

a) $\dfrac{1}{\sqrt{5}+\sqrt{2}}, \dfrac{1}{\sqrt{6}+\sqrt{2}}, \dfrac{1}{\sqrt{7}+\sqrt{2}}, \ldots$

$\dfrac{1}{\sqrt{5}+\sqrt{2}}$

$= \dfrac{1}{(\sqrt{5}+\sqrt{2})} \cdot \dfrac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5}-\sqrt{2})}$

$= \dfrac{\sqrt{5}-\sqrt{2}}{3}$

$\dfrac{1}{\sqrt{7}+\sqrt{2}}$

$= \dfrac{1}{(\sqrt{7}+\sqrt{2})} \cdot \dfrac{(\sqrt{7}-\sqrt{2})}{(\sqrt{7}-\sqrt{2})}$

$= \dfrac{\sqrt{7}-\sqrt{2}}{5}$

$\dfrac{1}{\sqrt{6}+\sqrt{2}}$

$= \dfrac{1}{(\sqrt{6}+\sqrt{2})} \cdot \dfrac{(\sqrt{6}-\sqrt{2})}{(\sqrt{6}-\sqrt{2})}$

$= \dfrac{\sqrt{6}-\sqrt{2}}{4}$

The numerator of the quotient is the conjugate of the denominator in the original expression. The denominator is the absolute value of the difference of the radicands. The next three quotients are:

$\dfrac{\sqrt{8}-\sqrt{2}}{6}, \dfrac{\sqrt{9}-\sqrt{2}}{7}, \dfrac{\sqrt{10}-\sqrt{2}}{8}$

b) $\dfrac{\sqrt{3}}{\sqrt{3}+\sqrt{2}}, \dfrac{\sqrt{4}}{\sqrt{4}+\sqrt{3}}, \dfrac{\sqrt{5}}{\sqrt{5}+\sqrt{4}}, \ldots$

$$\frac{\sqrt{3}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}}{(\sqrt{3}+\sqrt{2})} \cdot \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} = 3-\sqrt{6}$$

$$\frac{\sqrt{4}}{\sqrt{4}+\sqrt{3}} = \frac{\sqrt{4}}{(\sqrt{4}+\sqrt{3})} \cdot \frac{(\sqrt{4}-\sqrt{3})}{(\sqrt{4}-\sqrt{3})} = 4-\sqrt{12}$$

$$\frac{\sqrt{5}}{\sqrt{5}+\sqrt{4}} = \frac{\sqrt{5}}{(\sqrt{5}+\sqrt{4})} \cdot \frac{(\sqrt{5}-\sqrt{4})}{(\sqrt{5}-\sqrt{4})} = 5-\sqrt{20}$$

The first term of the quotient is the square of the numerator in the original expression. The second term is the square root of the product of the radicands in the denominator of the original expression. The next three quotients are:
$6-\sqrt{30}$, $7-\sqrt{42}$, $8-\sqrt{56}$

12. The dimensions of the Parthenon of ancient Greece are thought to be based on the *golden rectangle*. The length of a golden rectangle is $\dfrac{2}{\sqrt{5}-1}$ times as long as its width.

a) Write the value of $\dfrac{2}{\sqrt{5}-1}$ to 3 decimal places.

Use a calculator. $\dfrac{2}{\sqrt{5}-1} \doteq 1.618$

b) Write the value of the reciprocal of $\dfrac{2}{\sqrt{5}-1}$ to 3 decimal places.

Reciprocal: $\dfrac{\sqrt{5}-1}{2} \doteq 0.618$

c) Compare the values in parts a and b. How are they related?

$1.618 - 0.618 = 1$; the values have a difference of 1.

d) Write an expression to represent the difference between $\dfrac{2}{\sqrt{5}-1}$ and its reciprocal. Simplify the expression. What do you notice?

A common denominator is: $2(\sqrt{5}-1)$

$$\begin{aligned}\frac{2}{\sqrt{5}-1} - \frac{\sqrt{5}-1}{2} &= \frac{2(2)}{2(\sqrt{5}-1)} - \frac{(\sqrt{5}-1)(\sqrt{5}-1)}{2(\sqrt{5}-1)}\\ &= \frac{4-(5-\sqrt{5}-\sqrt{5}+1)}{2\sqrt{5}-2}\\ &= \frac{2\sqrt{5}-2}{2\sqrt{5}-2}\\ &= 1\end{aligned}$$

The expression simplifies to 1, which is the same as the difference between the estimated decimal values in part c.

C

13. a) Simplify.

i) $\dfrac{1}{\sqrt{5}} - \dfrac{1}{\sqrt{3}}$

A common denominator is: $\sqrt{15}$

$$\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{3}} = \frac{1 \cdot \sqrt{3}}{\sqrt{5} \cdot \sqrt{3}} - \frac{1 \cdot \sqrt{5}}{\sqrt{3} \cdot \sqrt{5}}$$
$$= \frac{\sqrt{3} - \sqrt{5}}{\sqrt{15}}$$
$$= \frac{(\sqrt{3} - \sqrt{5})}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}}$$
$$= \frac{\sqrt{3} \cdot \sqrt{15} - \sqrt{5} \cdot \sqrt{15}}{(\sqrt{15})^2}$$
$$= \frac{\sqrt{45} - \sqrt{75}}{15}$$
$$= \frac{3\sqrt{5} - 5\sqrt{3}}{15}$$

ii) $\dfrac{\sqrt{2}}{\sqrt{12}} - \dfrac{5\sqrt{3}}{\sqrt{8}}$

Simplify first.

$$\frac{\sqrt{2}}{\sqrt{12}} - \frac{5\sqrt{3}}{\sqrt{8}} = \frac{\sqrt{2}}{2\sqrt{3}} - \frac{5\sqrt{3}}{2\sqrt{2}}$$

A common denominator is: $2\sqrt{6}$

$$\frac{\sqrt{2}}{2\sqrt{3}} - \frac{5\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2} \cdot \sqrt{2}}{2\sqrt{3} \cdot \sqrt{2}} - \frac{5\sqrt{3} \cdot \sqrt{3}}{2\sqrt{2} \cdot \sqrt{3}}$$
$$= \frac{2 - 15}{2\sqrt{6}}$$
$$= \frac{-13}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$
$$= \frac{-13\sqrt{6}}{12}$$

iii) $\dfrac{\sqrt{6}}{2\sqrt{5} + 3\sqrt{3}} - \dfrac{\sqrt{2}}{\sqrt{7} - 2\sqrt{3}}$

Rationalize the denominator of each term, then subtract.

$$\frac{\sqrt{6}}{2\sqrt{5} + 3\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{7} - 2\sqrt{3}}$$
$$= \frac{\sqrt{6}}{(2\sqrt{5} + 3\sqrt{3})} \cdot \frac{(2\sqrt{5} - 3\sqrt{3})}{(2\sqrt{5} - 3\sqrt{3})} - \frac{\sqrt{2}}{(\sqrt{7} - 2\sqrt{3})} \cdot \frac{(\sqrt{7} + 2\sqrt{3})}{(\sqrt{7} + 2\sqrt{3})}$$
$$= \frac{2\sqrt{30} - 3\sqrt{18}}{20 - 27} - \frac{\sqrt{14} + 2\sqrt{6}}{7 - 12}$$
$$= \frac{2\sqrt{30} - 9\sqrt{2}}{-7} - \frac{\sqrt{14} + 2\sqrt{6}}{-5}$$
$$= \frac{-2\sqrt{30} + 9\sqrt{2}}{7} + \frac{\sqrt{14} + 2\sqrt{6}}{5} \quad \text{Common denominator: 35}$$
$$= \frac{5(-2\sqrt{30}) + 5(9\sqrt{2}) + 7\sqrt{14} + 7(2\sqrt{6})}{35}$$
$$= \frac{-10\sqrt{30} + 45\sqrt{2} + 7\sqrt{14} + 14\sqrt{6}}{35}$$

b) How are the strategies you used in part a similar to those used to add and subtract quotients of integers? How are they different?

The strategies are similar in that I still have to find a common denominator to add or subtract the terms. The strategies are different in that I have to rationalize the denominators.

14. Rationalize the denominator.

a) $\dfrac{3}{\sqrt[3]{4}}$

To make the denominator an integer, I multiply $\sqrt[3]{4}$ by $(\sqrt[3]{4})^2$.

$$\frac{3}{\sqrt[3]{4}} = \frac{3}{\sqrt[3]{4}} \cdot \frac{(\sqrt[3]{4})^2}{(\sqrt[3]{4})^2}$$
$$= \frac{3(\sqrt[3]{4})^2}{4}$$

b) $\dfrac{4}{\sqrt[4]{3}}$

To make the denominator an integer, I multiply $\sqrt[4]{3}$ by $(\sqrt[4]{3})^3$.

$$\frac{4}{\sqrt[4]{3}} = \frac{4}{\sqrt[4]{3}} \cdot \frac{(\sqrt[4]{3})^3}{(\sqrt[4]{3})^3}$$
$$= \frac{4(\sqrt[4]{3})^3}{3}$$

15. Write $\dfrac{2\sqrt{x} + 3\sqrt{y}}{\sqrt{x} - \sqrt{y}}$, $x, y \geq 0$ in simplest form.

$$\frac{2\sqrt{x} + 3\sqrt{y}}{\sqrt{x} - \sqrt{y}} = \frac{(2\sqrt{x} + 3\sqrt{y})}{(\sqrt{x} - \sqrt{y})} \cdot \frac{(\sqrt{x} + \sqrt{y})}{(\sqrt{x} + \sqrt{y})}$$
$$= \frac{2\sqrt{x}(\sqrt{x} + \sqrt{y}) + 3\sqrt{y}(\sqrt{x} + \sqrt{y})}{(\sqrt{x})^2 - (\sqrt{y})^2}$$
$$= \frac{2x + 2\sqrt{xy} + 3\sqrt{xy} + 3y}{x - y}$$
$$= \frac{2x + 5\sqrt{xy} + 3y}{x - y}$$

16. Decide if it is possible to determine two whole numbers a and b that satisfy each condition. Justify your answers.

a) $\sqrt{a} \cdot \sqrt{b}$ and $\dfrac{\sqrt{a}}{\sqrt{b}}$ are rational.

Yes, it is possible. When a and b are perfect squares, $b \neq 0$, $\sqrt{a} \cdot \sqrt{b}$ is a product of natural numbers and $\dfrac{\sqrt{a}}{\sqrt{b}}$ is a quotient of natural numbers. For example, $\sqrt{4} \cdot \sqrt{9} = 2 \cdot 3$, or 6, and $\dfrac{\sqrt{4}}{\sqrt{9}} = \dfrac{2}{3}$.

b) $\sqrt{a} \cdot \sqrt{b}$ and $\dfrac{\sqrt{a}}{\sqrt{b}}$ are irrational.

Yes, it is possible. When a and b are different prime numbers, $\sqrt{a} \cdot \sqrt{b}$ and $\dfrac{\sqrt{a}}{\sqrt{b}}$ are irrational. For example, $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$ or 2.4494 . . . , and $\dfrac{\sqrt{2}}{\sqrt{3}} = 0.8164 \ldots.$