Lesson 2.5 Exercises, pages 145–152

Students should verify the solutions to all equations.

A

4. Solve each equation. Verify the solution.

a) $\sqrt{3x} = 6, x \geq 0$

$$(\sqrt{3x})^2 = 6^2$$
$$3x = 36$$
$$x = 12$$

b) $2\sqrt{5x} = 10, x \geq 0$

$$\sqrt{5x} = 5$$
$$(\sqrt{5x})^2 = 5^2$$
$$5x = 25$$
$$x = 5$$

c) $42 = 7\sqrt{2x}, x \geq 0$

$$6 = \sqrt{2x}$$
$$(6)^2 = (\sqrt{2x})^2$$
$$36 = 2x$$
$$x = 18$$

d) $-2\sqrt{6x} = -12, x \geq 0$

$$\sqrt{6x} = 6$$
$$(\sqrt{6x})^2 = 6^2$$
$$6x = 36$$
$$x = 6$$

5. Solve each equation. Verify the solution.

a) $\sqrt{x - 2} = 5$

$x - 2 \geq 0$; that is, $x \geq 2$

$$(\sqrt{x - 2})^2 = 5^2$$
$$x - 2 = 25$$
$$x = 27$$

b) $\sqrt{3x + 1} = 5$

$3x + 1 \geq 0$; that is, $x \geq -\frac{1}{3}$

$$(\sqrt{3x + 1})^2 = 5^2$$
$$3x + 1 = 25$$
$$3x = 24$$
$$x = 8$$

c) $4 = \sqrt{2 - 7x}$

$2 - 7x \geq 0$; that is, $x \leq \frac{2}{7}$

$$4^2 = (\sqrt{2 - 7x})^2$$
$$16 = 2 - 7x$$
$$14 = -7x$$
$$x = -2$$

d) $3 = \sqrt{2x + 1}$

$2x + 1 \geq 0$; that is, $x \geq -\frac{1}{2}$

$$3^2 = (\sqrt{2x + 1})^2$$
$$9 = 2x + 1$$
$$8 = 2x$$
$$x = 4$$

e) $\sqrt{5x - 9} - 2 = 7$

$5x - 9 \geq 0$; that is, $x \geq \frac{9}{5}$

$\sqrt{5x - 9} = 9$

$(\sqrt{5x - 9})^2 = (9)^2$

$5x - 9 = 81$

$5x = 90$

$x = 18$

f) $2\sqrt{1 - 3x} + 1 = 9$

$1 - 3x \geq 0$; that is, $x \leq \frac{1}{3}$

$2\sqrt{1 - 3x} + 1 = 9$

$2\sqrt{1 - 3x} = 8$

$\sqrt{1 - 3x} = 4$

$(\sqrt{1 - 3x})^2 = (4)^2$

$1 - 3x = 16$

$-3x = 15$

$x = -5$

6. Determine whether the given value of x is a root of the equation.

a) $\sqrt{2x + 1} = 9$; $x = 16$

L.S. $= \sqrt{2(16)} + 1$ R.S. $= 9$

$= \sqrt{32} + 1$

$= 4\sqrt{2} + 1$

Since L.S. $\neq$ R.S., $x = 16$ is not a root of the equation.

b) $\sqrt{3x - 6} = 6$; $x = 14$

L.S. $= \sqrt{3(14) - 6}$ R.S. $= 6$

$= \sqrt{36}$

$= 6$

Since L.S. $=$ R.S., $x = 14$ is a root of the equation.

c) $9 = \sqrt{121 - 2x}$; $x = 20$

L.S. $= 9$ R.S. $= \sqrt{121 - 2(20)}$

$= \sqrt{81}$

$= 9$

Since L.S. $=$ R.S., $x = 20$ is a root of the equation.

d) $\sqrt{2x - 5} = \sqrt{3x - 2}$; $x = -3$

Since $2x - 5 \geq 0$, then $x \geq \frac{5}{2}$

Since $3x - 2 \geq 0$, then $x \geq \frac{2}{3}$

Since $x = -3$ does not lie in the set of possible values for x, $x = -3$ is not a root.

e) $\sqrt{2x + 5} = \sqrt{3x + 2}$; $x = 3$

L.S. $= \sqrt{2(3) + 5}$ R.S. $= \sqrt{3(3) + 2}$

$= \sqrt{11}$ $= \sqrt{11}$

Since L.S. $=$ R.S., $x = 3$ is a root of the equation.

B

7. Determine the root of each equation. Verify the solution.

a) $\sqrt{6x} = 2, x \geq 0$

$$\sqrt{6x} = 2$$
$$(\sqrt{6x})^2 = 2^2$$
$$6x = 4$$
$$x = \frac{4}{6}, \text{ or } \frac{2}{3}$$

b) $2 = \sqrt{2x + 1}$

$2x + 1 \geq 0$; that is, $x \geq -\frac{1}{2}$

$$2 = \sqrt{2x + 1}$$
$$2^2 = (\sqrt{2x + 1})^2$$
$$4 = 2x + 1$$
$$3 = 2x$$
$$x = \frac{3}{2}$$

c) $3 - \sqrt{x} = -2, x \geq 0$

$$-\sqrt{x} = -5$$
$$\sqrt{x} = 5$$
$$(\sqrt{x})^2 = 5^2$$
$$x = 25$$

d) $4 = \sqrt{-2x} + 3$

$-2x \geq 0$; that is, $x \leq 0$

$$4 = \sqrt{-2x} + 3$$
$$1 = \sqrt{-2x}$$
$$1^2 = (\sqrt{-2x})^2$$
$$1 = -2x$$
$$x = -\frac{1}{2}$$

e) $1 - 3\sqrt{5x} = -3 - 2\sqrt{5x}$

$x \geq 0$

$$1 - 3\sqrt{5x} = -3 - 2\sqrt{5x}$$
$$4 = \sqrt{5x}$$
$$4^2 = (\sqrt{5x})^2$$
$$16 = 5x$$
$$x = \frac{16}{5}, \text{ or } 3\frac{1}{5}$$

f) $2 - 2\sqrt{3x} = 1 - \sqrt{3x}$

$x \geq 0$

$$2 - 2\sqrt{3x} = 1 - \sqrt{3x}$$
$$1 = \sqrt{3x}$$
$$1^2 = (\sqrt{3x})^2$$
$$1 = 3x$$
$$x = \frac{1}{3}$$

8. The formula $V = \sqrt{PR}$ relates the potential difference across an electrical circuit, V volts, to the power, P watts, and the resistance, R ohms. The potential difference across a 40-W amplifier is 80 V. What is the resistance of the amplifier? How did you find out?

$V = \sqrt{PR}$ Substitute: $V = 80$, $P = 40$

$$80 = \sqrt{40R}$$
$$(80)^2 = (\sqrt{40R})^2$$
$$6400 = 40R$$
$$R = 160$$

The resistance of the amplifier is 160 ohms.

9. Determine the root of each equation. Verify the solution.

a) $\frac{\sqrt{x}}{5} = 2$

$x \geq 0$

$$\frac{\sqrt{x}}{5} = 2$$
$$\sqrt{x} = 10$$
$$(\sqrt{x})^2 = 10^2$$
$$x = 100$$

b) $-2 = \frac{-\sqrt{2x}}{4}$

$x \geq 0$

$$-2 = \frac{-\sqrt{2x}}{4}$$
$$-8 = -\sqrt{2x}$$
$$8 = \sqrt{2x}$$
$$8^2 = (\sqrt{2x})^2$$
$$64 = 2x$$
$$x = 32$$

c) $\frac{\sqrt{3x - 2}}{2} = 1$

$3x - 2 \geq 0$; that is, $x \geq \frac{2}{3}$

$$\frac{\sqrt{3x - 2}}{2} = 1$$
$$\sqrt{3x - 2} = 2$$
$$(\sqrt{3x - 2})^2 = 2^2$$
$$3x - 2 = 4$$
$$3x = 6$$
$$x = 2$$

d) $\sqrt{x - 7} = \frac{\sqrt{2x + 4}}{2}$

$x - 7 \geq 0$; that is, $x \geq 7$
$2x + 4 \geq 0$; that is, $x \geq -2$
So, for both radicals to be defined, $x \geq 7$

$$\sqrt{x - 7} = \frac{\sqrt{2x + 4}}{2}$$
$$2\sqrt{x - 7} = \sqrt{2x + 4}$$
$$(2\sqrt{x - 7})^2 = (\sqrt{2x + 4})^2$$
$$4(x - 7) = 2x + 4$$
$$4x - 28 = 2x + 4$$
$$2x = 32$$
$$x = 16$$

10. Which of these equations have real roots? Justify your answers.

a) $2 = \sqrt{3x - 1}$

$3x - 1 \geq 0$; that is, $x \geq \frac{1}{3}$

$$2 = \sqrt{3x - 1}$$
$$2^2 = (\sqrt{3x - 1})^2$$
$$4 = 3x - 1$$
$$5 = 3x$$
$$x = \frac{5}{3}$$

$x = \frac{5}{3}$ lies in the set of possible values for x. So, the equation has a real root.

b) $\sqrt{3x - 1} + 5 = 2$

$3x - 1 \geq 0$; that is, $x \geq \frac{1}{3}$

$$\sqrt{3x - 1} + 5 = 2$$
$$\sqrt{3x - 1} = -3$$

The left side of the equation is greater than or equal to 0.
The right side of the equation is negative, -3.
So, no real solutions are possible.
The equation has no real roots.

c) $\sqrt{5x + 3} = \sqrt{3x + 1}$

$5x + 3 \geq 0$; that is, $x \geq -\frac{3}{5}$

$3x + 1 \geq 0$; that is, $x \geq -\frac{1}{3}$

So, for both radicals to be defined, $x \geq -\frac{1}{3}$

$$\begin{aligned} \sqrt{5x + 3} &= \sqrt{3x + 1} \\ (\sqrt{5x + 3})^2 &= (\sqrt{3x + 1})^2 \\ 5x + 3 &= 3x + 1 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

Since $x = -1$ does not lie in the set of possible values for x, the equation has no real roots.

d) $\sqrt{-3x + 7} = \sqrt{-2x + 9}$

$-3x + 7 \geq 0$; that is, $x \leq \frac{7}{3}$

$-2x + 9 \geq 0$; that is, $x \leq \frac{9}{2}$

So, for both radicals to be defined, $x \leq \frac{7}{3}$

$$\begin{aligned} \sqrt{-3x + 7} &= \sqrt{-2x + 9} \\ (\sqrt{-3x + 7})^2 &= (\sqrt{-2x + 9})^2 \\ -3x + 7 &= -2x + 9 \\ -2 &= x \end{aligned}$$

Since $x = -2$ lies in the set of possible values for x, the equation has a real root.

11. The period, P seconds, of a pendulum is the time to complete one full swing. The period can be determined using the formula $P = 2\pi\sqrt{\frac{L}{9.8}}$, where L is the length of the pendulum in metres. Approximately how long should a pendulum be to complete one full swing in 2 s?

$P = 2\pi\sqrt{\frac{L}{9.8}}$ Substitute: $P = 2$

$$\begin{aligned} 2 &= 2\pi\sqrt{\frac{L}{9.8}} \\ \frac{1}{\pi} &= \sqrt{\frac{L}{9.8}} \\ \left(\frac{1}{\pi}\right)^2 &= \left(\sqrt{\frac{L}{9.8}}\right)^2 \\ \frac{1}{\pi^2} &= \frac{L}{9.8} \\ \frac{9.8}{\pi^2} &= L \\ L &= 0.9929\ldots \end{aligned}$$

The pendulum should be about 1 m long.

12. Which of these equations have real roots? Justify your answers.

a) $2\sqrt{x + 8} = 3\sqrt{3x + 1}$

$x + 8 \geq 0$; that is, $x \geq -8$

$3x + 1 \geq 0$; that is, $x \geq -\frac{1}{3}$

So, for both radicals to be defined, $x \geq -\frac{1}{3}$

$$\begin{aligned} 2\sqrt{x + 8} &= 3\sqrt{3x + 1} \\ (2\sqrt{x + 8})^2 &= (3\sqrt{3x + 1})^2 \\ 4(x + 8) &= 9(3x + 1) \\ 4x + 32 &= 27x + 9 \\ 23x &= 23 \\ x &= 1 \end{aligned}$$

$x = 1$ lies in the set of possible values for x. So, the equation has a real root.

b) $2\sqrt{x - 8} = 3\sqrt{3x + 1}$

$x - 8 \geq 0$; that is, $x \geq 8$

$3x + 1 \geq 0$; that is, $x \geq -\frac{1}{3}$

So, for both radicals to be defined, $x \geq 8$

$$\begin{aligned} 2\sqrt{x - 8} &= 3\sqrt{3x + 1} \\ (2\sqrt{x - 8})^2 &= (3\sqrt{3x + 1})^2 \\ 4(x - 8) &= 9(3x + 1) \\ 4x - 32 &= 27x + 9 \\ 23x &= -41 \\ x &= -\frac{41}{23} \end{aligned}$$

Since $x = -\frac{41}{23}$ does not lie in the set of possible values for x, the equation has no real roots.

c) $2\sqrt{x + 8} = 3\sqrt{3x - 1}$

$x + 8 \geq 0$; that is, $x \geq -8$

$3x - 1 \geq 0$; that is, $x \geq \frac{1}{3}$

So, for both radicals to be defined, $x \geq \frac{1}{3}$

$$\begin{aligned} 2\sqrt{x + 8} &= 3\sqrt{3x - 1} \\ (2\sqrt{x + 8})^2 &= (3\sqrt{3x - 1})^2 \\ 4(x + 8) &= 9(3x - 1) \\ 4x + 32 &= 27x - 9 \\ 23x &= 41 \\ x &= \frac{41}{23} \end{aligned}$$

$x = \frac{41}{23}$ lies in the set of possible values for x. So, the equation has a real root.

d) $2\sqrt{x - 8} = 3\sqrt{3x - 1}$

$x - 8 \geq 0$; that is, $x \geq 8$

$3x - 1 \geq 0$; that is, $x \geq \frac{1}{3}$

So, for both radicals to be defined, $x \geq 8$

$$\begin{aligned} 2\sqrt{x - 8} &= 3\sqrt{3x - 1} \\ (2\sqrt{x - 8})^2 &= (3\sqrt{3x - 1})^2 \\ 4(x - 8) &= 9(3x - 1) \\ 4x - 32 &= 27x - 9 \\ 23x &= -23 \\ x &= -1 \end{aligned}$$

Since $x = -1$ does not lie in the set of possible values for x, the equation has no real roots.

13. A student solved the equation $2\sqrt{5 - 4x} = -4$ and determined that the root was $x = \frac{1}{4}$. Is the student correct? If the student is correct, explain why. If the student is incorrect, what is the correct solution?

$5 - 4x \geq 0$; that is, $x \leq \frac{5}{4}$

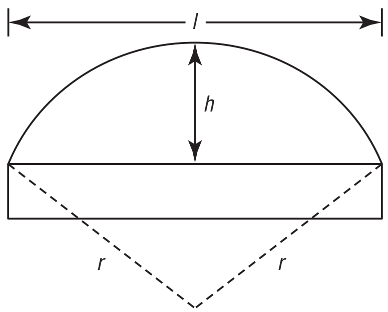
$$\begin{aligned} 2\sqrt{5 - 4x} &= -4 \\ \sqrt{5 - 4x} &= -2 \end{aligned}$$

The left side of the equation is greater than or equal to 0.
The right side of the equation is negative, -2.
So, no real solutions are possible.
The equation has no real roots.
So, the student is incorrect.

14. Write three radical equations that have a root of 4. Describe your strategy.

Start with the solution $x = 4$.
Add the same number to both sides so that the number on the right side of the equation is a perfect square: $5 + x = 4 + 5$
$5 + x = 9$
Then take the square root of both sides: $\sqrt{5 + x} = \sqrt{9}$
Use mental math to check. The equation has a root of 4.
Similarly, the equations $\sqrt{12 + x} = \sqrt{16}$ and $\sqrt{-3 + x} = \sqrt{1}$ have a root of 4.

15. The roof of a circular stadium is part of a sphere.
The formula $l = \sqrt{4h(2r - h)}$ relates the length of the stadium, l metres, to the maximum height, h metres, of the roof and the radius, r metres, of the sphere.



A stadium has length 160 m and maximum height 40 m.
What is the radius of the sphere?

$l = \sqrt{4h(2r - h)}$ Substitute: $l = 160$, $h = 40$

$160 = \sqrt{4(40)(2r - 40)}$

$160 = \sqrt{320r - 6400}$

$(160)^2 = (\sqrt{320r - 6400})^2$

$25\,600 = 320r - 6400$

$320r = 32\,000$

$r = 100$

The radius of the sphere is 100 m.

C

16. Earth approximates a sphere with radius 6370 km.

a) The formula for the surface area of a sphere is: $SA = 4\pi r^2$

To the nearest kilometre, determine the edge length of a cube with the same surface area as Earth.

$SA = 4\pi r^2$ Substitute: $r = 6370$

$SA = 4\pi(6370)^2$

$SA \doteq 509\,904\,363.8$

The surface area of Earth is approximately 509 904 363.8 km^2.

The formula for the surface area of a cube with edge length *e* kilometres is:

$SA = 6e^2$ Substitute: $SA = 509\,904\,363.8$

$509\,904\,363.8 = 6e^2$

$84\,984\,060.63 = e^2$

$\sqrt{84\,984\,060.63} = e$

$9218.6799\ldots = e$

The edge length of the cube is about 9219 km.

b) The formula for the volume of a sphere is: $V = \frac{4}{3}\pi r^3$

To the nearest kilometre, determine the edge length of a cube with the same volume as Earth.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 6370$

$V = \frac{4}{3}\pi(6370)^3$

$V \doteq 1.082\,696\,932 \times 10^{12}$

The volume of Earth is approximately $1.082\,696\,932 \times 10^{12}$ km^3.

The formula for the volume of a cube with edge length *e* kilometres is:

$V = e^3$ Substitute: $V = 1.082\,696\,932 \times 10^{12}$

$1.082\,696\,932 \times 10^{12} = e^3$

$\sqrt[3]{1.082\,696\,932 \times 10^{12}} = e$

$10\,268.388\,75\ldots = e$

The edge length of the cube is about 10 268 km.

17. Determine the root of each equation. Verify the solution. What strategy did you use?

a) $4 = \sqrt[3]{8x}$

$4^3 = (\sqrt[3]{8x})^3$

$64 = 8x$

$x = 8$

b) $\sqrt[3]{2x - 5} = 3$

$(\sqrt[3]{2x - 5})^3 = 3^3$

$2x - 5 = 27$

$2x = 32$

$x = 16$

c) $2 = \sqrt[3]{2x + 3} + 5$

$-3 = \sqrt[3]{2x + 3}$

$(-3)^3 = (\sqrt[3]{2x + 3})^3$

$-27 = 2x + 3$

$2x = -30$

$x = -15$

I isolated the radical, then cubed both sides of the equation to eliminate the radical sign. I then solved the equation for *x*, and verified my solution.