PRACTICE TEST, pages 162–164

1. Multiple Choice Which statement is false?

A. $|x| = \sqrt{x^2}$ for $x \in \mathbb{R}$

B. $x = \sqrt{x^2}$ for $x \geq 0$

(C.) $|x| = \sqrt[3]{x^3}$ for $x \in \mathbb{R}$

D. $x = \sqrt[3]{x^3}$ for $x \geq 0$

2. Multiple Choice Which is the correct simplification of $\sqrt{12x^3}$?

A. $2\sqrt{3x^3}, x \geq 0$

B. $2x\sqrt{3x}, x \in \mathbb{R}$

(C.) $2x\sqrt{3x}, x \geq 0$

D. $2|x|\sqrt{3x}, x \in \mathbb{R}$

3. Simplify.

a) $\sqrt{72} - 5\sqrt{2} + 3\sqrt{8}$

$= \sqrt{36 \cdot 2} - 5\sqrt{2} + 3\sqrt{4 \cdot 2}$
$= 6\sqrt{2} - 5\sqrt{2} + 6\sqrt{2}$
$= 7\sqrt{2}$

b) $(\sqrt{3} - \sqrt{5})(\sqrt{3} + \sqrt{5})$

$= \sqrt{3}(\sqrt{3} + \sqrt{5}) - \sqrt{5}(\sqrt{3} + \sqrt{5})$
$= 3 + \sqrt{15} - \sqrt{15} - 5$
$= -2$

c) $(2\sqrt{6} + 3\sqrt{5})^2$

$= (2\sqrt{6} + 3\sqrt{5})(2\sqrt{6} + 3\sqrt{5})$
$= 2\sqrt{6}(2\sqrt{6} + 3\sqrt{5})$
$+ 3\sqrt{5}(2\sqrt{6} + 3\sqrt{5})$
$= 24 + 6\sqrt{30} + 6\sqrt{30} + 45$
$= 69 + 12\sqrt{30}$

d) $\frac{1}{\sqrt{7} - \sqrt{3}}$

$= \frac{1}{(\sqrt{7} - \sqrt{3})} \cdot \frac{(\sqrt{7} + \sqrt{3})}{(\sqrt{7} + \sqrt{3})}$

$= \frac{\sqrt{7} + \sqrt{3}}{(\sqrt{7})^2 - (\sqrt{3})^2}$

$= \frac{\sqrt{7} + \sqrt{3}}{4}$

4. Identify the values of the variable for which each radical is defined where necessary, then simplify.

a) $(\sqrt{x} + 2)(\sqrt{x} - 3)$

The radicands cannot be negative, so $x \geq 0$.
$(\sqrt{x} + 2)(\sqrt{x} - 3)$
$= \sqrt{x}(\sqrt{x} - 3) + 2(\sqrt{x} - 3)$
$= x - 3\sqrt{x} + 2\sqrt{x} - 6$
$= x - \sqrt{x} - 6$

b) $6\sqrt{a^2} + 2a$

$\sqrt{a^2} \in \mathbb{R}$ when $a^2 \geq 0$.
$a^2 \geq 0$, so $\sqrt{a^2}$ is defined for $a \in \mathbb{R}$.
$6\sqrt{a^2} + 2a$
$= 6|a| + 2a$

c) $\dfrac{8\sqrt{2}}{\sqrt{12} - \sqrt{10}}$

$= \dfrac{8\sqrt{2}}{2\sqrt{3} - \sqrt{10}}$

$= \dfrac{8\sqrt{2}}{(2\sqrt{3} - \sqrt{10})} \cdot \dfrac{(2\sqrt{3} + \sqrt{10})}{(2\sqrt{3} + \sqrt{10})}$

$= \dfrac{8\sqrt{2}(2\sqrt{3}) + 8\sqrt{2}(\sqrt{10})}{(2\sqrt{3})^2 - (\sqrt{10})^2}$

$= \dfrac{16\sqrt{6} + 8\sqrt{20}}{12 - 10}$

$= \dfrac{16\sqrt{6} + 16\sqrt{5}}{2}$

$= 8\sqrt{6} + 8\sqrt{5}$

d) $\dfrac{2\sqrt{10} - \sqrt{3}}{\sqrt{10} + \sqrt{3}}$

$= \dfrac{(2\sqrt{10} - \sqrt{3})}{(\sqrt{10} + \sqrt{3})} \cdot \dfrac{(\sqrt{10} - \sqrt{3})}{(\sqrt{10} - \sqrt{3})}$

$= \dfrac{2\sqrt{10}(\sqrt{10} - \sqrt{3}) - \sqrt{3}(\sqrt{10} - \sqrt{3})}{(\sqrt{10})^2 - (\sqrt{3})^2}$

$= \dfrac{20 - 2\sqrt{30} - \sqrt{30} + 3}{10 - 3}$

$= \dfrac{23 - 3\sqrt{30}}{7}$

5. Which equations have real roots? If the root is real, determine its value. If the equation has no real roots, explain how you know.

a) $\sqrt{2x + 3} = 3$

$2x + 3 \geq 0$; that is, $x \geq -\frac{3}{2}$

$\sqrt{2x + 3} = 3$

$(\sqrt{2x + 3})^2 = 3^2$

$2x + 3 = 9$

$2x = 6$

$x = 3$

$x = 3$ lies in the set of possible values for x. So, the equation has a real root.

b) $\sqrt{5x - 1} = \sqrt{2x + 5}$

$5x - 1 \geq 0$; that is, $x \geq \frac{1}{5}$

$2x + 5 \geq 0$; that is, $x \geq -\frac{5}{2}$

So, for both radicals to be defined, $x \geq \frac{1}{5}$

$\sqrt{5x - 1} = \sqrt{2x + 5}$

$(\sqrt{5x - 1})^2 = (\sqrt{2x + 5})^2$

$5x - 1 = 2x + 5$

$3x = 6$

$x = 2$

$x = 2$ lies in the set of possible values for x. So, the equation has a real root.

c) $\sqrt{3x + 2} + 5 = 2$

$3x + 2 \geq 0$; that is, $x \geq -\frac{2}{3}$

$\sqrt{3x + 2} + 5 = 2$

$\sqrt{3x + 2} = -3$

The left side of the equation is greater than or equal to 0. The right side of the equation is negative, -3.
So, no real solutions are possible.
The equation has no real roots.

d) $2\sqrt{x - 8} = 3\sqrt{x + 2}$

$x - 8 \geq 0$; that is, $x \geq 8$

$x + 2 \geq 0$; that is, $x \geq -2$

So, for both radicals to be defined, $x \geq 8$

$2\sqrt{x - 8} = 3\sqrt{x + 2}$

$(2\sqrt{x - 8})^2 = (3\sqrt{x + 2})^2$

$4(x - 8) = 9(x + 2)$

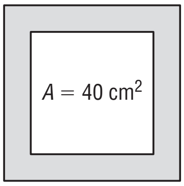
$4x - 32 = 9x + 18$

$-50 = 5x$

$x = -10$

$x = -10$ does not lie in the set of possible values for x. So, the equation has no real roots.

6. To make a picture frame, a square with area 40 cm^2 is cut from a square with area 90 cm^2. Serena wants to put a thin gold ribbon around the inside and outside edges of the frame. How much ribbon does Serena need?



The side length of a square is the square root of its area.

So, the side length of the square with area 40 cm^2 is:

$\sqrt{40} = 2\sqrt{10}$ cm

The side length of the square with area 90 cm^2 is:

$\sqrt{90} = 3\sqrt{10}$ cm

The length of ribbon needed = perimeter of large square + perimeter of small square

$$= 4(2\sqrt{10}) + 4(3\sqrt{10})$$
$$= 8\sqrt{10} + 12\sqrt{10}$$
$$= 20\sqrt{10}$$

Serena needs $20\sqrt{10}$ cm of ribbon.

7. The formula $t = \sqrt{\frac{2d}{9.8}}$ gives the time, t seconds, for an object at rest to fall d metres. It took 2.5 s for a ball dropped from a roof to hit the ground. To the nearest metre, from what height was the ball dropped?

$t = \sqrt{\frac{2d}{9.8}}$ Substitute: $t = 2.5$

$$2.5 = \sqrt{\frac{2d}{9.8}}$$
$$(2.5)^2 = \left(\sqrt{\frac{2d}{9.8}}\right)^2$$
$$6.25 = \frac{2d}{9.8}$$
$$61.25 = 2d$$
$$d = 30.625$$

The ball was dropped from a height of about 31 m.