# Lesson 3.1 Exercises, pages 176–182

## A

**3.** Factor.

**a)** $x^2 - 100$

$= (x)^2 - (10)^2$
$= (x + 10)(x - 10)$

**b)** $49x^2 - 1$

$= (7x)^2 - (1)^2$
$= (7x + 1)(7x - 1)$

**c)** $4 - 49n^2$

$= (2)^2 - (7n)^2$
$= (2 + 7n)(2 - 7n)$

**d)** $64x^2 - 25y^2$

$= (8x)^2 - (5y)^2$
$= (8x - 5y)(8x + 5y)$

**4.** Factor.

**a)** $x^2 + 7x + 10$

Find 2 numbers with sum 7 and product 10.
$= (x + 2)(x + 5)$

**b)** $x^2 - 11x + 30$

Find 2 numbers with sum $-11$ and product 30.
$= (x - 5)(x - 6)$

**c)** $m^2 + 4m - 45$

Find 2 numbers with sum 4 and product $-45$.
$= (m + 9)(m - 5)$

**d)** $y^2 - 2y - 63$

Find 2 numbers with sum $-2$ and product $-63$.
$= (y - 9)(y + 7)$

**5.** Factor.

**a)** $5x^2 + 9x + 4$

Guess and test factors of 5 with factors of 4.
$= (5x + 4)(x + 1)$

**b)** $4x^2 - 7x + 3$

Guess and test factors of 4 with negative factors of 3.
$= (x - 1)(4x - 3)$

**c)** $6x^2 + x - 2$

Guess and test factors of 6 with factors of $-2$.
$= (3x + 2)(2x - 1)$

**d)** $2x^2 - 9x + 4$

Guess and test factors of 2 with negative factors of 4.
$= (x - 4)(2x - 1)$

**6.** Factor.

**a)** $9x^2 + 6x + 1$

Guess and test factors of 9 with factors of 1.
$= (3x + 1)(3x + 1)$,
or $(3x + 1)^2$

**b)** $25x^2 + 20x + 4$

This is a perfect square trinomial. It can be written as: $(5x)^2 + 2(5x)(2) + 2^2$
$= (5x + 2)^2$

**c)** $36x^2 - 60x + 25$

This is a perfect square trinomial. It can be written as: $(6x)^2 + 2(6x)(-5) + (-5)^2$
$= (6x - 5)^2$

**d)** $4y^2 - 20y + 25$

This is a perfect square trinomial. It can be written as: $(2y)^2 + 2(2y)(-5) + (-5)^2$
$= (2y - 5)^2$

## B

**7.** Determine whether $x + 5$ is a factor of each polynomial.

**a)** $2x^2 - 2x - 40$

Write the trinomial as:
$(x + 5)(2x + b) =$
$2x^2 + (b + 10)x + 5b$
Equate constant terms.
$5b = -40$, so $b = -8$
Check:
$(x + 5)(2x - 8) =$
$2x^2 + 2x - 40$
So, $x + 5$ is not a factor.

**b)** $3x^2 + 13x - 10$

Write the trinomial as:
$(x + 5)(3x + b) =$
$3x^2 + (b + 15)x + 5b$
Equate constant terms.
$5b = -10$, so $b = -2$
Check:
$(x + 5)(3x - 2) =$
$3x^2 + 13x - 10$
So, $x + 5$ is a factor.

**8.** Determine whether $2x + 1$ is a factor of each polynomial.

**a)** $6x^2 - 13x - 8$

Write the trinomial as:
$(2x + 1)(3x + b) =$
$6x^2 + (2b + 3)x + b$
Equate constant terms.
$b = -8$
Check:
$(2x + 1)(3x - 8) =$
$6x^2 - 13x - 8$
So, $2x + 1$ is a factor.

**b)** $2x^2 - 8$

Write the binomial as:
$(2x + 1)(x + b) =$
$2x^2 + (2b + 1)x + b$
Equate constant terms.
$b = -8$
Check:
$(2x + 1)(x - 8) =$
$2x^2 - 15x - 8$
So, $2x + 1$ is not a factor.

**9.** Factor.

**a)** $2x^2 - 50y^2$

$= 2(x^2 - 25y^2)$
$= 2(x + 5y)(x - 5y)$

**b)** $0.1x^2 - 0.001$

$= 0.1(x^2 - 0.01)$
$= 0.1(x + 0.1)(x - 0.1)$

**c)** $20x^2 - 125y^2$

$= 5(4x^2 - 25y^2)$
$= 5(2x + 5y)(2x - 5y)$

**d)** $\frac{1}{100}x^2 - \frac{1}{25}y^2$

$= \frac{1}{25}\left(\frac{1}{4}x^2 - y^2\right)$
$= \frac{1}{25}\left(\frac{1}{2}x + y\right)\left(\frac{1}{2}x - y\right)$

**10.** Factor.

**a)** $2x^2 + 16x + 24$

$= 2(x^2 + 8x + 12)$
Find 2 numbers with sum 8 and product 12.
$= 2(x + 2)(x + 6)$

**b)** $3x^2 - 9x - 30$

$= 3(x^2 - 3x - 10)$
Find 2 numbers with sum $-3$ and product $-10$.
$= 3(x - 5)(x + 2)$

**c)** $x^2 + \frac{5}{2}x - 6$

$= \frac{1}{2}(2x^2 + 5x - 12)$
Guess and test factors of 2 with factors of $-12$.
$= \frac{1}{2}(2x - 3)(x + 4)$

**d)** $x^2 + 2.5x - 1.5$

$= 0.5(2x^2 + 5x - 3)$
Guess and test factors of 2 with factors of $-3$.
$= 0.5(2x - 1)(x + 3)$

**11.** Factor each polynomial.

**a)** $\frac{x^2}{9} - \frac{4}{25}$

$= \left(\frac{x}{3}\right)^2 - \left(\frac{2}{5}\right)^2$

$= \left(\frac{x}{3} + \frac{2}{5}\right)\left(\frac{x}{3} - \frac{2}{5}\right)$

**b)** $6 + 5x - x^2$

Guess and test factors of 6 with factors of $-1$.

$= (6 - x)(1 + x)$

**c)** $-x^2 + \frac{121}{64}$

$= \left(\frac{11}{8}\right)^2 - (x)^2$

$= \left(\frac{11}{8} + x\right)\left(\frac{11}{8} - x\right)$

**d)** $7 - \frac{5}{3}x - 2x^2$

$= \frac{1}{3}(21 - 5x - 6x^2)$

Use decomposition.

$= \frac{1}{3}(21 - 14x + 9x - 6x^2)$

$= \frac{1}{3}[7(3 - 2x) + 3x(3 - 2x)]$

$= \frac{1}{3}(3 - 2x)(7 + 3x)$

**12.** Factor each polynomial expression.

**a)** **i)** $9x^2 - 4y^2$

$= (3x + 2y)(3x - 2y)$

**ii)** $9(x - 3)^2 - 4(2y + 1)^2$

$= [3(x - 3)]^2 - [2(2y + 1)]^2$
$= [3(x - 3) + 2(2y + 1)][3(x - 3) - 2(2y + 1)]$
$= [3x - 9 + 4y + 2][3x - 9 - 4y - 2]$
$= (3x + 4y - 7)(3x - 4y - 11)$

**b)** **i)** $50x^2 - 162y^2$

$= 2(25x^2 - 81y^2)$
$= 2(5x + 9y)(5x - 9y)$

**ii)** $50(2x - 5)^2 - 162(3y - 2)^2$

$= 2[25(2x - 5)^2 - 81(3y - 2)^2]$
$= 2([5(2x - 5)]^2 - [9(3y - 2)]^2)$
$= 2[5(2x - 5) + 9(3y - 2)][5(2x - 5) - 9(3y - 2)]$
$= 2[10x - 25 + 27y - 18][10x - 25 - 27y + 18]$
$= 2(10x + 27y - 43)(10x - 27y - 7)$

**13.** Factor each polynomial expression.

**a)** $16(2x - 7)^2 - 25(y + 2)^2$

$= [4(2x - 7)]^2 - [5(y + 2)]^2$
$= [4(2x - 7) + 5(y + 2)][4(2x - 7) - 5(y + 2)]$
$= (8x - 28 + 5y + 10)(8x - 28 - 5y - 10)$
$= (8x + 5y - 18)(8x - 5y - 38)$

**b)** $121(x + 3)^2 - 36(2y - 5)^2$

$= [11(x + 3)]^2 - [6(2y - 5)]^2$
$= [11(x + 3) + 6(2y - 5)][11(x + 3) - 6(2y - 5)]$
$= (11x + 33 + 12y - 30)(11x + 33 - 12y + 30)$
$= (11x + 12y + 3)(11x - 12y + 63)$

**14.** Use two different strategies to factor this polynomial expression: $(2x + 3)^2 - (x - 5)^2$. Which strategy is more efficient?

**Strategy 1:**

$(2x + 3)^2 - (x - 5)^2 = [(2x + 3) + (x - 5)][(2x + 3) - (x - 5)]$
$= (2x + 3 + x - 5)(2x + 3 - x + 5)$
$= (3x - 2)(x + 8)$

**Strategy 2:**

$(2x + 3)^2 - (x - 5)^2 = (2x + 3)(2x + 3) - (x - 5)(x - 5)$
$= (4x^2 + 12x + 9) - (x^2 - 10x + 25)$
$= 4x^2 + 12x + 9 - x^2 + 10x - 25$
$= 3x^2 + 22x - 16$
$= (3x - 2)(x + 8)$

**Strategy 1 is more efficient.**

**15.** Factor each polynomial expression.

**a) i)** $3x^2 + 19x + 16$

Guess and test factors of 3 with factors of 16.
$= (3x + 16)(x + 1)$

**ii)** $3(2x - 1)^2 + 19(2x - 1) + 16$

Use the factors in part i.
$= [3(2x - 1) + 16][(2x - 1) + 1]$
$= [6x - 3 + 16][2x - 1 + 1]$
$= (6x + 13)(2x)$

**b) i)** $12x^2 + 17x - 5$

Guess and test factors of 12 with factors of $-5$.
$= (4x - 1)(3x + 5)$

**ii)** $12(4x - 1)^2 + 17(4x - 1) - 5$

Use the factors in part i.
$= [4(4x - 1) - 1][3(4x - 1) + 5]$
$= [16x - 4 - 1][12x - 3 + 5]$
$= (16x - 5)(12x + 2)$
$= 2(16x - 5)(6x + 1)$

**16.** Factor each polynomial expression.

**a)** $(7x - 5)^2 - 8(7x - 5) + 15$

Find 2 numbers with sum $-8$ and product 15.
$= [(7x - 5) - 3][(7x - 5) - 5]$
$= (7x - 5 - 3)(7x - 5 - 5)$
$= (7x - 8)(7x - 10)$

**b)** $9(2x + 1)^2 - 42(2x + 1) + 49$

Guess and test factors of 9 with negative factors of 49.
$= [3(2x + 1) - 7][3(2x + 1) - 7]$
$= (6x + 3 - 7)(6x + 3 - 7)$
$= (6x - 4)(6x - 4)$
$= 4(3x - 2)^2$

**c)** $(x^2 - x)^2 - 8(x^2 - x) + 12$

Find 2 numbers with sum $-8$ and product 12.
$= [(x^2 - x) - 2][(x^2 - x) - 6]$
$= (x^2 - x - 2)(x^2 - x - 6)$
$= (x - 2)(x + 1)(x - 3)(x + 2)$

**d)** $(4x^2 + 4x + 3)^2 - 8(4x^2 + 4x + 3) + 12$

Find 2 numbers with sum $-8$ and product 12.
$= [(4x^2 + 4x + 3) - 2][(4x^2 + 4x + 3) - 6]$
$= (4x^2 + 4x + 3 - 2)(4x^2 + 4x + 3 - 6)$
$= (4x^2 + 4x + 1)(4x^2 + 4x - 3)$
$= (2x + 1)(2x + 1)(2x - 1)(2x + 3)$
$= (2x + 1)^2(2x - 1)(2x + 3)$

## C

**17.** Is $x + 1$ a factor of each polynomial expression?

**a)** $(x + 1)^2 + 4(x + 1) - 32$

Factor. Find 2 numbers with sum 4 and product $-32$.
$= [(x + 1) + 8][(x + 1) - 4]$
$= (x + 9)(x - 3)$
So, $x + 1$ is not a factor.

**b)** $2(2x + 3)^2 + 5(2x + 3) - 7$

Factor. Use guess and test with factors of 2 and factors of $-7$.
$= [2(2x + 3) + 7][(2x + 3) - 1]$
$= (4x + 13)(2x + 2)$
$= 2(4x + 13)(x + 1)$
So, $x + 1$ is a factor.

**18.** Factor each polynomial.

**a)** $x^2 + x + \frac{1}{4}$

Find two numbers with sum 1 and product $\frac{1}{4}$.
$= \left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$, or $\left(x + \frac{1}{2}\right)^2$

**b)** $x^2 + \frac{3}{2}x + \frac{1}{2}$

Find two numbers with sum $\frac{3}{2}$ and product $\frac{1}{2}$.
$= (x + 1)\left(x + \frac{1}{2}\right)$

**c)** $\frac{x^2}{2} - \frac{x}{4} - \frac{3}{4}$

$= \frac{1}{2}\left(x^2 - \frac{1}{2}x - \frac{3}{2}\right)$
Find two numbers with sum $-\frac{1}{2}$ and product $-\frac{3}{2}$.
$= \frac{1}{2}(x + 1)\left(x - \frac{3}{2}\right)$

**d)** $\frac{4}{3}x^2 - \frac{1}{12}y^2 = \frac{1}{3}\left(4x^2 - \frac{1}{4}y^2\right)$
$= \frac{1}{3}\left(2x + \frac{1}{2}y\right)\left(2x - \frac{1}{2}y\right)$

**19.** Factor each polynomial. Explain your strategy.

**a)** $x^2 - 6x + 9 - y^2$

The first 3 terms are a perfect square trinomial.
$= (x - 3)^2 - y^2$
Use the difference of squares.
$= [(x - 3) + y][(x - 3) - y]$
$= (x + y - 3)(x - y - 3)$

**b)** $4x^2 + 4x + 1 - 9y^2$

The first 3 terms are a perfect square trinomial.
$= (2x + 1)^2 - 9y^2$
Use the difference of squares.
$= [(2x + 1) + 3y][(2x + 1) - 3y]$
$= (2x + 3y + 1)(2x - 3y + 1)$

**20.** Consider the polynomial $3x^2 + nx - 4$. Determine a value for $n$ so that $3x - 2$ is a factor of the polynomial.

Write $3x^2 + nx - 4 = (3x - 2)(x + a)$
Equate the constant terms.
If $3x - 2$ is a factor, then $-4 = -2a$, so $a = 2$
Substitute this value for $a$ in the right side above, then expand.
$(3x - 2)(x + 2) = 3x^2 + 4x - 4$
The value of $n$ is 4.