Lesson 3.2 Exercises, pages 190–195

Students should verify all the solutions.

A

4. Which equations are quadratic equations? Explain how you know.

a) $3x^2 = 30$

This equation is quadratic because the variable term with the greatest exponent is an x^2-term.

b) $x^2 - 9x + 8 = 0$

This equation is quadratic because it is written in the form $ax^2 + bx + c = 0$.

c) $x^3 - x^2 + 5 = 0$

This equation is not quadratic; it cannot be written in the form $ax^2 + bx + c = 0$ because it has one x^3-term.

d) $6x + 5 = x - 7$

This equation is not quadratic; it cannot be written in the form $ax^2 + bx + c = 0$ because it does not have an x^2-term.

5. Solve each quadratic equation. Verify the solutions.

a) $(x + 5)(x + 8) = 0$

Either $x + 5 = 0$, then $x = -5$; or $x + 8 = 0$, then $x = -8$

b) $(x - 1)(x - 10) = 0$

Either $x - 1 = 0$, then $x = 1$; or $x - 10 = 0$, then $x = 10$

c) $(2x - 3)(x + 6) = 0$

Either $2x - 3 = 0$
$2x = 3$
$x = 1.5$
or $x + 6 = 0$, then $x = -6$

d) $(3x + 2)(x - 5) = 0$

Either $3x + 2 = 0$
$3x = -2$
$x = -\frac{2}{3}$
or $x - 5 = 0$, then $x = 5$

6. Solve.

a) $4(x + 5)(x + 9) = 0$

Either $x + 5 = 0$, then $x = -5$; or $x + 9 = 0$, then $x = -9$

b) $3x(x + 4) = 0$

Either $3x = 0$, then $x = 0$; or $x + 4 = 0$, then $x = -4$

c) $x(x - 4) = 0$

Either $x = 0$;
or $x - 4 = 0$, then $x = 4$

d) $5(2x - 1)(3x + 7) = 0$

Either $2x = 1$, then $x = \frac{1}{2}$;
or $3x + 7 = 0$, then $x = -\frac{7}{3}$

B

7. Solve by factoring. Verify the solutions.

a) $x^2 - 6x + 5 = 0$

$(x - 1)(x - 5) = 0$
Either $x - 1 = 0$, then $x = 1$;
or $x - 5 = 0$, then $x = 5$

b) $3x^2 - 21x - 54 = 0$

$3(x^2 - 7x - 18) = 0$
$3(x - 9)(x + 2) = 0$
Either $x - 9 = 0$, then $x = 9$;
or $x + 2 = 0$, then $x = -2$

c) $2x^2 - 15x + 25 = 0$

$(2x - 5)(x - 5) = 0$
Either $2x - 5 = 0$,
then $x = 2.5$;
or $x - 5 = 0$, then $x = 5$

d) $10x^2 + x - 3 = 0$

$(2x - 1)(5x + 3) = 0$
Either $2x - 1 = 0$, then $x = \frac{1}{2}$;
or $5x + 3 = 0$, then $x = -\frac{3}{5}$

8. Solve by factoring.

a) $x^2 - 6x = 27$

$x^2 - 6x - 27 = 0$
$(x - 9)(x + 3) = 0$
Either $x - 9 = 0$, then $x = 9$;
or $x + 3 = 0$, then $x = -3$

b) $3x^2 - 4x = 7$

$3x^2 - 4x - 7 = 0$
$(3x - 7)(x + 1) = 0$
Either $3x - 7 = 0$, then $x = \frac{7}{3}$;
or $x + 1 = 0$, then $x = -1$

c) $x^2 - 8x + 12 = 12$

$x^2 - 8x = 0$
$x(x - 8) = 0$
Either $x = 0$;
or $x - 8 = 0$, then $x = 8$

d) $3x^2 - 6x = 105$

$3x^2 - 6x - 105 = 0$
$3(x^2 - 2x - 35) = 0$
$3(x - 7)(x + 5) = 0$
Either $x - 7 = 0$, then $x = 7$;
or $x + 5 = 0$, then $x = -5$

9. A student wrote the solution below to solve this quadratic equation:
$(x - 5)(x + 2) = 8$

Either $x - 5 = 2$ or $x + 2 = 4$

$x = 7$ $\quad$ $x = 2$

Identify the error, then write the correct solution.

One side of the equation must be 0 before factoring.

$(x - 5)(x + 2) = 8$
$x^2 - 3x - 10 - 8 = 0$
$x^2 - 3x - 18 = 0$
$(x + 3)(x - 6) = 0$
Either $x + 3 = 0$ or $x - 6 = 0$
$x = -3$ $\quad$ $x = 6$

10. Solve each equation.

a) $(x + 3)(x + 4) = 6$

$x^2 + 7x + 12 - 6 = 0$
$x^2 + 7x + 6 = 0$
$(x + 1)(x + 6) = 0$
Either $x + 1 = 0$, then $x = -1$; or $x + 6 = 0$, then $x = -6$

b) $x^2 - 9 = 4x + 36$

$x^2 - 9 - 4x - 36 = 0$
$x^2 - 4x - 45 = 0$
$(x + 5)(x - 9) = 0$
Either $x + 5 = 0$, then $x = -5$; or $x - 9 = 0$, then $x = 9$

c) $3x^2 + 6 = x(x + 13)$

$3x^2 + 6 = x^2 + 13x$
$2x^2 - 13x + 6 = 0$
$(2x - 1)(x - 6) = 0$
Either $2x - 1 = 0$, then $x = 0.5$; or $x - 6 = 0$, then $x = 6$

d) $2x(x - 6) + 3x = 2x - 9$

$2x^2 - 12x + 3x - 2x + 9 = 0$
$2x^2 - 11x + 9 = 0$
$(2x - 9)(x - 1) = 0$
Either $2x - 9 = 0$, then $x = 4.5$; or $x - 1 = 0$, then $x = 1$

11. Create a quadratic equation that has exactly one root. Explain your strategy.

Sample response: Work backward. Choose a root of $x = 6$.
The equation is: $(x - 6)(x - 6) = 0$
Expand: $x^2 - 12x + 36 = 0$

12. Solve each equation, then verify the solution:

a) $\sqrt{23 - x} = x - 3$

$(\sqrt{23 - x})^2 = (x - 3)^2$
$23 - x = x^2 - 6x + 9$
$0 = x^2 - 5x - 14$
$0 = (x - 7)(x + 2)$
Either $x - 7 = 0$, then $x = 7$; or $x + 2 = 0$, then $x = -2$
I used mental math to verify.
$x \neq -2$; the root is $x = 7$

b) $\sqrt{2x^2 + 1} + 1 = 2x$

$\sqrt{2x^2 + 1} = 2x - 1$
$(\sqrt{2x^2 + 1})^2 = (2x - 1)^2$
$2x^2 + 1 = 4x^2 - 4x + 1$
$0 = 2x^2 - 4x$
$0 = 2x(x - 2)$
Either $2x = 0$, then $x = 0$; or $x - 2 = 0$, then $x = 2$
I used mental math to verify.
$x \neq 0$; the root is $x = 2$

13. Solve this equation: $\frac{x^2}{2} + \frac{7x}{6} = 1$

Multiply each term by the common denominator 6.
$3x^2 + 7x = 6$
$3x^2 + 7x - 6 = 0$
$(3x - 2)(x + 3) = 0$
Either $3x - 2 = 0$, $x = \frac{2}{3}$ or $x + 3 = 0$, $x = -3$

14. A football is kicked vertically. The approximate height of the football, h metres, after t seconds is modelled by this formula:
$h = 1 + 20t - 5t^2$

a) Determine the height of the football after 2 s.

$h = 1 + 20t - 5t^2$ Substitute $t = 2$.
$h = 1 + 20(2) - 5(2)^2$
$h = 21$
The football is 21 m high.

b) When is the football 16 m high?

$h = 1 + 20t - 5t^2$ Substitute $h = 16$, then solve for t.
$16 = 1 + 20t - 5t^2$
$5t^2 - 20t + 15 = 0$
$5(t^2 - 4t + 3) = 0$
$5(t - 1)(t - 3) = 0$
So, $t = 1$ or $t = 3$
The football is 16 m high after 1 s and after 3 s.

c) Why are there two solutions for part b?

The football is 16 m high as it is rising and as it is falling.

15. A student wrote the solution below to solve this quadratic equation:
$5x^2 = 25x$

$$\frac{25x^2}{5x} = \frac{25x}{5x}$$
$$x = 5$$

Identify the error, then write the correct solution.

The student should not have divided by x since $x = 0$ is a solution.
$5x^2 = 25x$
$5x^2 - 25x = 0$
$5x(x - 5) = 0$
Either $5x = 0$ or $x - 5 = 0$
$x = 0$ $x = 5$
The roots are: $x = 0$ and $x = 5$

16. When twice a number is subtracted from the square of the number, the result is 99. Determine the number.

Let x represent the number.
An equation is: $x^2 - 2x = 99$
$x^2 - 2x - 99 = 0$
$(x - 11)(x + 9) = 0$
Either $x - 11 = 0$ or $x + 9 = 0$
$x = 11$ $x = -9$
The numbers are 11 and -9.

17. Cody has a daily reading program where he reads for 2 min more than he did the day before. On the first day, he read for 10 min. Cody continued his program until he had read for a total of 6 h.

a) Write an equation to represent the number of days, n, for which Cody read.

The time, in minutes, for which Cody reads is:
$10 + 12 + 14 + \ldots$
This is an arithmetic series with 1st term 10 and common difference 2.
The total time Cody reads, 6 h or 360 min, is the sum of this series.

Use: $S_n = \frac{n[2t_1 + d(n - 1)]}{2}$ Substitute: $S_n = 360$, $t_1 = 10$, $d = 2$

$360 = \frac{n[2(10) + 2(n - 1)]}{2}$
$720 = n(20 + 2n - 2)$
$720 = n(18 + 2n)$
$720 = 18n + 2n^2$
$0 = 2n^2 + 18n - 720$ Divide each term by 2.
$0 = n^2 + 9n - 360$

b) Solve the equation for n.

$n^2 + 9n - 360 = 0$
$(n + 24)(n - 15) = 0$
Either $n + 24 = 0$ or $n - 15 = 0$
$n = -24$ $n = 15$
Since the number of days cannot be negative, Cody read for 15 days.

C

18. Solve each equation, then verify the solutions.

a) $(2x + 1)^2 = (x + 5)^2$

$4x^2 + 4x + 1 = x^2 + 10x + 25$
$3x^2 - 6x - 24 = 0$ Divide each term by 3.
$x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
Either $x - 4 = 0$ or $x + 2 = 0$
$x = 4$ $x = -2$

b) $(2x - 1)^2 - 2(2x - 1) - 8 = 0$

This equation has the same form as the simplified equation in part a, where x is replaced with $2x - 1$.
$[(2x - 1) - 4][(2x - 1) + 2] = 0$
$(2x - 5)(2x + 1) = 0$
Either $2x - 5 = 0$ or $2x + 1 = 0$
$x = 2.5$ $x = -0.5$

19. The area of a rectangular sheep pen is 96 m^2. The pen is divided into two smaller pens by inserting a fence parallel to the width of the pen. A total of 48 m of fencing is used. Determine the dimensions of the pen.

The area of the rectangular pen is 96 m^2, and the width is w metres, so the length, in metres, is $\frac{96}{w}$.
The total length of fencing is 48 m, so an equation is:

$$3w + 2\left(\frac{96}{w}\right) = 48 \quad \text{Multiply each term by } w.$$
$$3w^2 + 2(96) = 48w$$
$$3w^2 - 48w + 192 = 0 \quad \text{Divide each term by 3.}$$
$$w^2 - 16w + 64 = 0$$
$$(w - 8)(w - 8) = 0$$
$$w - 8 = 0$$
$$w = 8$$

The pen is 8 m wide, so its length is $\frac{96}{8}$ m, or 12 m.

20. A rectangular garden has dimensions 3 m by 4 m. A path is built around the garden. The area of the garden and path is 6 times as great as the area of the garden. What is the width of the path?

Let the width of the path be x metres.
Then the width of the rectangle formed by the garden and path, in metres, is $3 + 2x$, and the length of this rectangle, in metres, is $4 + 2x$.
The area of the garden is 12 m^2.
The areas of the garden and path, in square metres, is:
$(3 + 2x)(4 + 2x)$
An equation is: $(3 + 2x)(4 + 2x) = 6(12)$

$$12 + 14x + 4x^2 = 72$$
$$4x^2 + 14x - 60 = 0 \quad \text{Divide each term by 2.}$$
$$2x^2 + 7x - 30 = 0$$
$$(2x - 5)(x + 6) = 0$$

Either $2x - 5 = 0$ or $x + 6 = 0$
$x = 2.5$ $\quad$ $x = -6$

Since the width of the path cannot be negative, the width is 2.5 m.