

Lesson 3.3 Exercises, pages 206–211

A

4. Solve each equation.

a) $x^2 = 49$

$$x = \pm\sqrt{49}$$

$$x = \pm 7$$

b) $(x + 4)^2 = 9$

$$x + 4 = \pm\sqrt{9}$$

$$x + 4 = \pm 3$$

Either $x + 4 = 3$,

then $x = -1$; or

$x + 4 = -3$, then $x = -7$

c) $(x - 1)^2 - 16 = 20$ d) $2x^2 - 8 = 40$

$(x - 1)^2 = 36$

$x - 1 = \pm\sqrt{36}$

$x - 1 = \pm 6$

Either $x - 1 = 6$, then $x = 7$;

or $x - 1 = -6$, then $x = -5$

$2x^2 = 48$

$x^2 = 24$

$x = \pm\sqrt{24}$

5. Determine the value of \square that makes each trinomial a perfect square, then factor the trinomial.

a) $x^2 + 14x + \square$

The 3rd term is: $\left(\frac{14}{2}\right)^2 = 49$

$x^2 + 14x + 49$
 $= (x + 7)(x + 7)$
 $= (x + 7)^2$

b) $x^2 - 8x + \square$

The 3rd term is: $\left(\frac{-8}{2}\right)^2 = 16$

$x^2 - 8x + 16 = (x - 4)(x - 4)$
 $= (x - 4)^2$

c) $x^2 - 11x + \square$

The 3rd term is:

$\left(\frac{-11}{2}\right)^2 = 30.25$

$x^2 - 11x + 30.25$
 $= (x - 5.5)^2$

d) $x^2 + 3x + \square$

The 3rd term is:

$\left(\frac{3}{2}\right)^2 = 2.25$

$x^2 + 3x + 2.25$
 $= (x + 1.5)^2$

B

6. When a hailstone falls, its approximate terminal velocity, v centimetres per second, can be modelled by the equation $v^2 = 2\,000\,000d$, where d is the diameter of the hailstone in centimetres. The largest hailstone ever recovered from a storm fell in Aurora, Nebraska, on June 22, 2003. Its diameter was 17.8 cm. Determine the approximate terminal velocity of this hailstone.

$v^2 = 2\,000\,000d$ Substitute: $d = 17.8$ cm

$v^2 = 2\,000\,000(17.8)$ Solve for v .

$v = \pm\sqrt{2\,000\,000(17.8)}$ Ignore the negative root.

$v = 5966.5735\dots$

The terminal velocity was approximately 5967 cm/s.

7. The volume of a cone with height h metres and radius r metres is given by the formula $V = \frac{1}{3}\pi r^2 h$. What is the radius of a cone with volume 8.8 m^3 and height 5.0 m ? Give the answer to the nearest tenth of a metre.

$$V = \frac{1}{3}\pi r^2 h \quad \text{Substitute: } V = 8.8, h = 5$$

$$8.8 = \frac{1}{3}\pi r^2(5)$$

$$26.4 = 5\pi r^2$$

$$r = \sqrt{\frac{26.4}{5\pi}}$$

$$r = 1.2964\dots$$

The radius of the cone is approximately 1.3 m.

8. Solve each equation by completing the square.

a) $x^2 + 4x = 2$

$$x^2 + 4x + 4 = 2 + 4$$

$$(x + 2)^2 = 6$$

$$x + 2 = \pm\sqrt{6}$$

$$x = -2 \pm \sqrt{6}$$

b) $x^2 - 2x = 1$

$$x^2 - 2x + 1 = 1 + 1$$

$$(x - 1)^2 = 2$$

$$x - 1 = \pm\sqrt{2}$$

$$x = 1 \pm \sqrt{2}$$

c) $x^2 - 7x + 11 = 0$

$$x^2 - 7x = -11$$

$$x^2 - 7x + 12.25 = -11 + 12.25$$

$$(x - 3.5)^2 = 1.25$$

$$x - 3.5 = \pm\sqrt{1.25}$$

$$x = 3.5 \pm \sqrt{1.25}$$

d) $x^2 - 9x - 4 = 0$

$$x^2 - 9x = 4$$

$$x^2 - 9x + 20.25 = 4 + 20.25$$

$$(x - 4.5)^2 = 24.25$$

$$x - 4.5 = \pm\sqrt{24.25}$$

$$x = 4.5 \pm \sqrt{24.25}$$

9. A student wrote the solution below to solve this quadratic equation:

$$x^2 + 10x + 4 = 0$$

$$x^2 + 10x + 4 = 0$$

$$x^2 + 10x = -4$$

$$x^2 + 10x + 100 = -4 + 100$$

$$(x + 10)^2 = 96$$

$$x + 10 = \pm\sqrt{96}$$

$$x = -10 \pm \sqrt{96}$$

The roots are: $x = -10 + \sqrt{96}$ and $x = -10 - \sqrt{96}$

Identify the error, then write the correct solution.

When the student completed the square, he did not square one-half the coefficient of x .

$$x^2 + 10x + 4 = 0$$

$$x^2 + 10x = -4$$

$$x^2 + 10x + 25 = -4 + 25$$

$$(x + 5)^2 = 21$$

$$x + 5 = \pm\sqrt{21}$$

$$x = -5 \pm \sqrt{21}$$

10. Solve each equation by completing the square.

a) $3x^2 + 18x - 2 = 0$

$$x^2 + 6x - \frac{2}{3} = 0$$

$$x^2 + 6x = \frac{2}{3}$$

$$x^2 + 6x + 9 = \frac{2}{3} + 9$$

$$(x + 3)^2 = \frac{29}{3}$$

$$x + 3 = \pm\sqrt{\frac{29}{3}}$$

$$x = -3 \pm \sqrt{\frac{29}{3}}$$

b) $\frac{1}{2}x^2 + 3x + 1 = 0$

$$x^2 + 6x + 2 = 0$$

$$x^2 + 6x = -2$$

$$x^2 + 6x + 9 = -2 + 9$$

$$(x + 3)^2 = 7$$

$$x + 3 = \pm\sqrt{7}$$

$$x = -3 \pm \sqrt{7}$$

c) $5x^2 - 20x + 8 = 0$

$$x^2 - 4x + 1.6 = 0$$

$$x^2 - 4x = -1.6$$

$$x^2 - 4x + 4 = -1.6 + 4$$

$$(x - 2)^2 = 2.4$$

$$x - 2 = \pm\sqrt{2.4}$$

$$x = 2 \pm \sqrt{2.4}$$

d) $-2x^2 + 16x - 3 = 0$

$$x^2 - 8x + 1.5 = 0$$

$$x^2 - 8x = -1.5$$

$$x^2 - 8x + 16 = -1.5 + 16$$

$$(x - 4)^2 = 14.5$$

$$x - 4 = \pm\sqrt{14.5}$$

$$x = 4 \pm \sqrt{14.5}$$

11. Solve each equation by completing the square.

a) $x^2 + 5x - 3 = 0$

$$x^2 + 5x = 3$$

$$x^2 + 5x + 6.25 = 3 + 6.25$$

$$x^2 + 5x + 6.25 = 9.25$$

$$(x + 2.5)^2 = 9.25$$

$$x + 2.5 = \pm\sqrt{9.25}$$

$$x = -2.5 \pm \sqrt{9.25}$$

b) $-3x^2 - 7x + 5 = 0$

$$-3x^2 - 7x + 5 = 0$$

$$x^2 + \frac{7}{3}x - \frac{5}{3} = 0$$

$$x^2 + \frac{7}{3}x = \frac{5}{3}$$

$$x^2 + \frac{7}{3}x + \frac{49}{36} = \frac{5}{3} + \frac{49}{36}$$

$$\left(x + \frac{7}{6}\right)^2 = \frac{109}{36}$$

$$x + \frac{7}{6} = \pm\sqrt{\frac{109}{36}}$$

$$x = -\frac{7}{6} \pm \sqrt{\frac{109}{36}}, \text{ or } \frac{-7 \pm \sqrt{109}}{6}$$

- 12.** When the square of a number is added to the number, the sum is 3. What is the number? Justify your answer.

Let the number be x .

An equation is: $x^2 + x = 3$ Complete the square.

$$x^2 + x + 0.25 = 3 + 0.25$$

$$(x + 0.5)^2 = 3.25$$

$$x + 0.5 = \pm\sqrt{3.25}$$

$$x = -0.5 \pm \sqrt{3.25}$$

There are two possible numbers: $-0.5 - \sqrt{3.25}$ and $-0.5 + \sqrt{3.25}$

C

- 13.** The following problem was first proposed by Jerome Cardan in 1545.

Divide 10 into two parts so that when one part is multiplied by the other, the product is 40.

Attempt to solve this problem by completing the square. Explain why there is no solution.

Let x represent one part, then the other part is $10 - x$.

An equation is: $x(10 - x) = 40$

$$10x - x^2 = 40$$

$$x^2 - 10x = -40$$

Complete the square.

$$x^2 - 10x + 25 = -40 + 25$$

$$(x - 5)^2 = -15$$

The left side is a perfect square and the right side is negative, so there are no real roots.

- 14.** Consider the quadratic equation $x^2 + bx + 10 = 0$, where b is a constant. Determine the possible values of b so that this equation does not have a solution. Explain your strategy.

$$x^2 + bx + 10 = 0$$

$$x^2 + bx = -10$$

Complete the square.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -10 + \left(\frac{b}{2}\right)^2$$

$$\left(x + \frac{b}{2}\right)^2 = -10 + \left(\frac{b}{2}\right)^2$$

For no real solution, the right side must be negative; that is,

$$-10 + \left(\frac{b}{2}\right)^2 < 0$$

$$\left(\frac{b}{2}\right)^2 < 10$$

$$b^2 < 40$$

$$b < \sqrt{40} \text{ and } b > -\sqrt{40}$$

So, $-\sqrt{40} < b < \sqrt{40}$

- 15.** Consider the quadratic equation $2x^2 - 6x + c = 0$, where c is a constant. Determine the possible values of c so that this equation has a real solution. Explain your strategy.

$$2x^2 - 6x + c = 0 \quad \text{Complete the square.}$$

$$x^2 - 3x + \frac{c}{2} = 0$$

$$x^2 - 3x + 2.25 = -\frac{c}{2} + 2.25$$

$$(x - 1.5)^2 = -\frac{c}{2} + 2.25$$

For a real solution, the right side must be positive or 0; that is,

$$-\frac{c}{2} + 2.25 \geq 0$$

$$c \leq 4.5$$