Lesson 4.1 Exercises, pages 257–261

When approximating answers, round to the nearest tenth.

A

4. Identify the y-intercept of the graph of each quadratic function.

a) $y = -\frac{1}{2}x^2 + 5x - 1$

b) $y = 3 - 14x + 5x^2$

Use mental math. Substitute $x = 0$.

a) $y = -1$

b) $y = 3$

c) $y = -4x + 3x^2$

d) $y = \frac{4}{3}x^2$

c) $y = 0$

d) $y = 0$

5. State whether the vertex of the graph of each quadratic function is a maximum point or a minimum point.

a) $y = 2x^2 + 5x - 4$

b) $y = 5 - 3x^2$

a) Coefficient of x^2 is positive.
So, parabola opens up.
Vertex is a minimum point.

b) Coefficient of x^2 is negative.
So, parabola opens down.
Vertex is a maximum point.

B

6. Identify whether each table of values represents a linear function, a quadratic function, or neither. Explain how you know.

a)

x	0	−1	−2	−3	−4
y	−3	−2	0	4	12

The x-coordinates decrease by 1 each time.
First differences:
$-2 - (-3) = 1$
$0 - (-2) = 2$
$4 - 0 = 4$
$12 - 4 = 8$
The first differences are not constant, and they do not increase or decrease by the same number. So the function is neither linear nor quadratic.

b)

x	0	2	4	6	8
y	5	0	−7	−16	−27

The x-coordinates increase by 2 each time.
First differences:
$0 - 5 = -5$
$-7 - 0 = -7$
$-16 - (-7) = -9$
$-27 - (-16) = -11$
The first differences decrease by 2 each time. So, the function is quadratic.

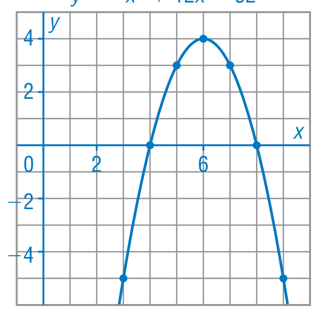
7. Use a table of values to graph each quadratic function below, for the values of x indicated. Determine:

i) the intercepts
ii) the coordinates of the vertex
iii) the equation of the axis of symmetry
iv) the domain of the function
v) the range of the function

a) $y = -x^2 + 12x - 32$

x	3	4	5	6	7	8	9
y	−5	0	3	4	3	0	−5

$y = -x^2 + 12x - 32$

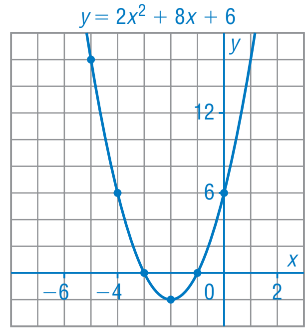


i) x-intercepts: 4, 8
from equation, y-intercept: −32
ii) vertex: (6, 4)
iii) axis of symmetry: $x = 6$
iv) domain: $x \in \mathbb{R}$
v) range: $y \leq 4$, $y \in \mathbb{R}$

b) $y = 2x^2 + 8x + 6$

x	−5	−4	−3	−2	−1	0
y	16	6	0	−2	0	6

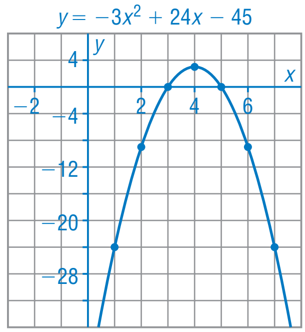
$y = 2x^2 + 8x + 6$



i) x-intercepts: −3, −1
y-intercept: 6
ii) vertex: (−2, −2)
iii) axis of symmetry: $x = -2$
iv) domain: $x \in \mathbb{R}$
v) range: $y \geq -2$, $y \in \mathbb{R}$

c) $y = -3x^2 + 24x - 45$

x	1	2	3	4	5	6	7
y	−24	−9	0	3	0	−9	−24



i) x-intercepts: 3, 5
from equation, y-intercept: −45
ii) vertex: (4, 3)
iii) axis of symmetry: $x = 4$
iv) domain: $x \in \mathbb{R}$
v) range: $y \leq 3, y \in \mathbb{R}$

8. a) Use a graphing calculator to graph each set of quadratic functions.

i) $y = x^2 + 2x$
$y = x^2 + 2x + 1$
$y = x^2 + 2x + 2$

ii) $y = -x^2 - 2x$
$y = -x^2 - 2x - 1$
$y = -x^2 - 2x - 2$

b) How many x-intercepts may a parabola have?

$y = x^2 + 2x$ has 2 x-intercepts; $y = x^2 + 2x + 1$ has 1 x-intercept; and $y = x^2 + 2x + 2$ has no x-intercepts.
$y = -x^2 - 2x$ has 2 x-intercepts; $y = -x^2 - 2x - 1$ has 1 x-intercept; and $y = -x^2 - 2x - 2$ has no x-intercepts.
So, a parabola may have 0, 1, or 2 x-intercepts.

c) How many y-intercepts may a parabola have?

Each of the quadratic functions in part a has 1 y-intercept.
So, a parabola may have 1 y-intercept.

9. Use a graphing calculator to graph each quadratic function below. Determine:

i) the intercepts
ii) the coordinates of the vertex
iii) the equation of the axis of symmetry
iv) the domain of the function
v) the range of the function

a) $y = 0.5x^2 - 2x + 5$
b) $y = -0.75x^2 + 6x - 15$

Once I had graphed a function, I used the CALC feature, where necessary, to determine the intercepts and the coordinates of the vertex. When the intercepts were approximate, I wrote them to the nearest hundredth.

a)
i) x-intercepts: none
y-intercept: 5
ii) vertex: (2, 3)
iii) axis of symmetry: $x = 2$
iv) domain: $x \in \mathbb{R}$
v) range: $y \geq 3, y \in \mathbb{R}$

b)
i) x-intercepts: none
y-intercept: −15
ii) vertex: (4, −3)
iii) axis of symmetry: $x = 4$
iv) domain: $x \in \mathbb{R}$
v) range: $y \leq -3, y \in \mathbb{R}$

c) $y = 2x^2 - 3x - 2.875$

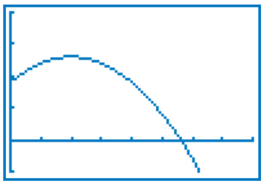
i) x-intercepts: about -0.66 and about 2.16
y-intercept: -2.875
ii) vertex: $(0.75, -4)$
iii) axis of symmetry: $x = 0.75$
iv) domain: $x \in \mathbb{R}$
v) range: $y \geq -4$, $y \in \mathbb{R}$

d) $y = -3x^2 + 10.5x - 8.1875$

i) x-intercepts: about 1.17 and about 2.33
y-intercept: -8.1875
ii) vertex: $(1.75, 1)$
iii) axis of symmetry: $x = 1.75$
iv) domain: $x \in \mathbb{R}$
v) range: $y \leq 1$, $y \in \mathbb{R}$

10. A stone is dropped from a bridge over the Peace River. The height of the stone, h metres, above the river, t seconds after it was dropped, is modelled by the equation $h = 20 - 4.9t^2$.

a) Graph the quadratic function, then sketch it below.



I input the equation $y = 20 - 4.9x^2$ in my graphing calculator, then graphed the function.

b) When did the stone hit the river?

To the nearest tenth, the positive t-intercept is 2.0. So, the stone hit the river approximately 2 s after it was dropped.

c) What is the domain? What does it represent?

The domain is the set of possible t-values; that is, all values between and including 0 and the positive t-intercept. To the nearest tenth of a second, the domain is: $0 \leq t \leq 2.0$. The domain represents the time the stone was in the air.

C

11. Consider the quadratic function $y = ax^2 + c$. What must be true about a and c in each case?

a) The function has no x-intercepts.

When a is positive, the parabola opens up. So, for the function to have no x-intercepts, its vertex must be above the x-axis; that is $c > 0$.
When a is negative, the parabola opens down. So, for the function to have no x-intercepts, its vertex must be below the x-axis; that is $c < 0$.

b) The function has one x-intercept.

For the function to have one x-intercept, the vertex of the parabola must be on the x-axis. So, $c = 0$

c) The function has two x-intercepts.

When a is positive, the parabola opens up. So, for the function to have two x-intercepts, its vertex must lie below the x-axis; that is $c < 0$. When a is negative, the parabola opens down. So, for the function to have two x-intercepts, its vertex must lie above the x-axis; that is $c > 0$.

12. In *Example* 3 on page 255, the parabola has one positive t-intercept which is approximately 12.3. The graph has also a negative t-intercept.

a) Determine this intercept.

Using the CALC feature on my graphing calculator, the negative t-intercept, to the nearest hundredth, is -0.03.

b) How could you interpret this intercept in terms of the situation modelled by the equation?

The object is fired from a location just above the ground. The numerical value of the negative t-intercept could represent the additional time that it would take the object to reach its maximum height if it were fired from the ground.