Lesson 4.4 Exercises, pages 284–289

A

3. Identify the coordinates of the vertex of the graph of each quadratic function.

a) $y = 3(x - 5)^2 + 4$

b) $y = 0.75(x - 0.25)^2 - 1.5$

Compare each equation with the standard form $y = a(x - p)^2 + q$.

$p = 5$ and $q = 4$, so the
coordinates of the vertex
are: (5, 4)

$p = 0.25$ and $q = -1.5$, so the
coordinates of the vertex
are: (0.25, −1.5)

c) $y = -2x^2 + 1$

d) $y = -\frac{1}{2}(x + 4)^2$

$p = 0$ and $q = 1$, so the
coordinates of the vertex
are: (0, 1)

$p = -4$ and $q = 0$, so the
coordinates of the vertex
are: (−4, 0)

4. Describe the transformation that would be applied to the graph of $y = x^2$ to get the graph of each function.
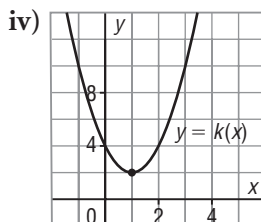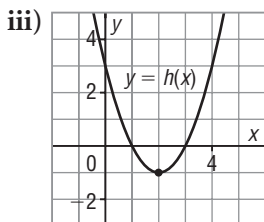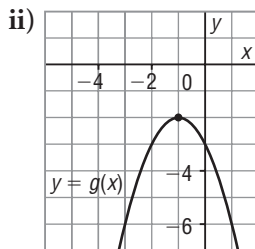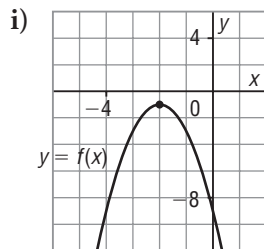
a) $y = (x - 3)^2$

b) $y = (x + 1)^2$

Compare with $y = (x - p)^2$.
$p = 3$, so the graph of $y = x^2$
would be translated 3 units right.

Compare with $y = (x - p)^2$.
$p = -1$, so the graph of $y = x^2$
would be translated 1 unit left.

5. Match each equation to a graph below.

i)

ii)

iii)

iv)

a) $y = -(x + 1)^2 - 2$ **b)** $y = 2(x - 1)^2 + 2$

Compare each equation with the standard form $y = a(x - p)^2 + q$.

a is negative, so the graph opens down. $p = -1$ and $q = -2$, so the coordinates of the vertex are: $(-1, -2)$ **This is graph ii.**	*a* is positive, so the graph opens up. $p = 1$ and $q = 2$, so the coordinates of the vertex are: $(1, 2)$ **This is graph iv.**

c) $y = -2(x + 2)^2 - 1$ **d)** $y = (x - 2)^2 - 1$

a is negative, so the graph opens down. $p = -2$ and $q = -1$, so the coordinates of the vertex are: $(-2, -1)$ **This is graph i.**	*a* is positive, so the graph opens up. $p = 2$ and $q = -1$, so the coordinates of the vertex are: $(2, -1)$ **This is graph iii.**

B

6. a) Identify the coordinates of the vertex of the graph of each quadratic function.

**i)** $y = -3x^2 + 2$ **ii)** $y = \frac{2}{5}(x + 1)^2$

Compare each equation with the standard form $y = a(x - p)^2 + q$.

$p = 0$ and $q = 2$, so the coordinates of the vertex are: $(0, 2)$	$p = -1$ and $q = 0$, so the coordinates of the vertex are: $(-1, 0)$

b) How can you use algebra to verify that the coordinates in part a are correct? How can you use a graphing calculator to verify?

I can substitute the values into the original equation. If the left side equals the right side, the coordinates are correct. I can enter the equation into my graphing calculator, then use the CALC feature to determine the coordinates of the vertex.

7. Determine an equation of the graph of each quadratic function.

a)

b)
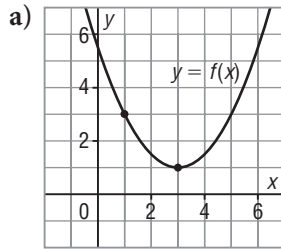
Use: $y = a(x - p)^2 + q$.
The coordinates of the
vertex are $(p, q) = (3, 1)$
Substitute the values of p
and q in $y = a(x - p)^2 + q$.
The equation becomes:
$y = a(x - 3)^2 + 1$.
Substitute the coordinates
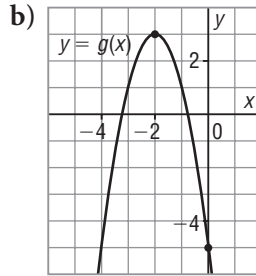of a point on the graph,
such as: $(1, 3)$
$3 = a(1 - 3)^2 + 1$
$3 = 4a + 1$
$2 = 4a$
$a = \frac{2}{4}$, or $\frac{1}{2}$
The equation is:
$y = \frac{1}{2}(x - 3)^2 + 1$

Use: $y = a(x - p)^2 + q$.
The coordinates of the vertex are
$(p, q) = (-2, 3)$
Substitute the values of p and q in
$y = a(x - p)^2 + q$.
The equation becomes:
$y = a(x + 2)^2 + 3$.
Substitute the coordinates of a point
on the graph,
such as: $(0, -5)$
$-5 = a(0 + 2)^2 + 3$
$-5 = 4a + 3$
$-8 = 4a$
$a = -2$
The equation is:
$y = -2(x + 2)^2 + 3$

8. For each quadratic function below:

$\quad$ **i)** $y = 3(x - 2)^2 + 1$ $\qquad$ **ii)** $y = -\frac{1}{4}(x + 8)^2 - 1$

a) Identify the coordinates of the vertex, the domain, the range, the
direction of opening, the equation of the axis of symmetry, and
the intercepts. If necessary, give the answers to the nearest tenth.

$\quad$ **i)** $y = 3(x - 2)^2 + 1$

a is positive, so the graph opens up.
$p = 2$ and $q = 1$, so the coordinates of the vertex are: $(2, 1)$
The equation of the axis of symmetry is $x = p$; that is, $x = 2$.
To determine the y-intercept, substitute $x = 0$:
$y = 3(0 - 2)^2 + 1$
$y = 13$
The y-intercept is 13.
To determine the x-intercepts, substitute $y = 0$:
$\quad 0 = 3(x - 2)^2 + 1$
$-\frac{1}{3} = (x - 2)^2$
This equation has no solution, so there are no x-intercepts.
The domain is: $x \in \mathbb{R}$
The graph opens up, so the vertex is a minimum point with
y-coordinate 1. The range is: $y \geq 1$, $y \in \mathbb{R}$

ii) $y = -\frac{1}{4}(x + 8)^2 - 1$

a is negative, so the graph opens down.
$p = -8$ and $q = -1$, so the coordinates of the vertex are: $(-8, -1)$
The equation of the axis of symmetry is $x = p$; that is, $x = -8$.
To determine the y-intercept, substitute $x = 0$:
$y = -\frac{1}{4}(0 + 8)^2 - 1$
$y = -17$
The y-intercept is -17.
To determine the x-intercepts, substitute $y = 0$:
$0 = -\frac{1}{4}(x + 8)^2 - 1$
$1 = -\frac{1}{4}(x + 8)^2$
$-4 = (x + 8)^2$
This equation has no solution, so there are no x-intercepts.
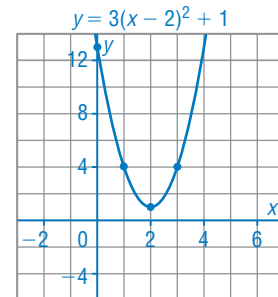The domain is: $x \in \mathbb{R}$
The graph opens down, so the vertex is a maximum point with
y-coordinate -1. The range is: $y \le -1, y \in \mathbb{R}$

b) Sketch a graph.
   i) $y = 3(x - 2)^2 + 1$

The graph is congruent to the graph of $y = 3x^2$.
On a grid, mark a point at the vertex $(2, 1)$. Use the step
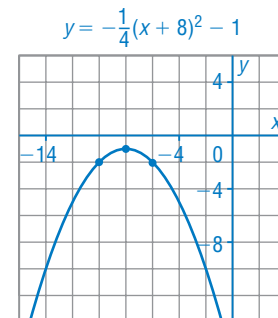pattern. Multiply each vertical step by 3.

$y = 3(x - 2)^2 + 1$

   ii) $y = -\frac{1}{4}(x + 8)^2 - 1$

The graph is congruent to the graph of $y = -\frac{1}{4}x^2$.

On a grid, mark a point at the vertex $(-8, -1)$.
Use the step pattern. Multiply each vertical step by -0.25.

$y = -\frac{1}{4}(x + 8)^2 - 1$

9. Determine an equation of a quadratic function with the given
characteristics of its graph.

   a) coordinates of the vertex: $V(2, -1)$; passes through $A(4, -3)$

An equation has the form $y = a(x - p)^2 + q$.
The vertex is at $V(2, -1)$, so $p = 2$ and $q = -1$.
The equation becomes $y = a(x - 2)^2 - 1$.
Substitute: $x = 4$, $y = -3$
$-3 = a(4 - 2)^2 - 1$
$-3 = 4a - 1$
$a = -\frac{1}{2}$
So, an equation of the function is: $y = -\frac{1}{2}(x - 2)^2 - 1$

b) coordinates of the vertex: $V(4, 5)$; y-intercept -43

An equation has the form $y = a(x - p)^2 + q$.
The vertex is at V(4, 5), so $p = 4$ and $q = 5$.
The equation becomes $y = a(x - 4)^2 + 5$.
Substitute the coordinates of the y-intercept: $(0, -43)$
$x = 0$, $y = -43$
$-43 = a(0 - 4)^2 + 5$
$-48 = 16a$
$\quad a = -3$
So, an equation of the function is: $y = -3(x - 4)^2 + 5$

10. Use a graphing calculator or graphing software.

a) Graph each quadratic function. How many x-intercepts does the graph have?

   i) $y = x^2$ **ii)** $y = 2(x - 5)^2 + 3$ **iii)** $y = \frac{3}{5}x^2 - 2$

   iv) $y = -\frac{1}{2}x^2 + 1$ **v)** $y = -4x^2 - 1$ **vi)** $y = -2(x + 3)^2$

i) one   ii) none   iii) two
iv) two   v) none   vi) one

b) How can you use the values of a and q in the equation $y = a(x - p)^2 + q$ to determine how many x-intercepts the graph of a quadratic function has?

When a is positive, the parabola opens up. So, for the function to have no x-intercepts, its vertex must lie above the x-axis; that is, $c > 0$. For the function to have one x-intercept, the vertex of the parabola must lie on the x-axis; that is, $c = 0$. For the function to have two x-intercepts, its vertex must lie below the x-axis; that is, $c < 0$.
When a is negative, the parabola opens down. So, for the function to have no x-intercepts, its vertex must lie below the x-axis; that is, $c < 0$. For the function to have one x-intercept, the vertex of the parabola must lie on the x-axis; that is, $c = 0$. For the function to have two x-intercepts, its vertex must lie above the x-axis; that is, $c > 0$.

c) Write an equation of a quadratic function, different from those in part a, whose graph has each number of x-intercepts:

   i) 0   **ii)** 1   **iii)** 2

How do you know that you are correct?

Sample response:
  i) $y = 3(x - 4)^2 + 7$ ii) $y = -5(x - 1)^2$ iii) $y = -4(x + 7)^2 + 9$
I graphed each quadratic function on my graphing calculator to check that the number of intercepts was correct.

11. At a Canada Day celebration in Surrey, B.C., a firework is projected vertically into the air and reaches a maximum height of 100 m. The path of the firework is parabolic and it lands 30 m from the launch site. Determine an equation that models the height of the firework as a function of the horizontal distance travelled.

Sample response: On the coordinate plane, let the x-axis represent the ground and the origin represent the launch site.
The firework lands 30 m from the launch site, so an x-intercept of the parabola is 30. The axis of symmetry is midway between $x = 0$ and $x = 30$, or $x = 15$. The maximum height is 100 m, so the vertex is at (15, 100).
An equation has the form $y = a(x - p)^2 + q$.
The vertex is at V(15, 100), so $p = 15$ and $q = 100$.
The equation becomes $y = a(x - 15)^2 + 100$.
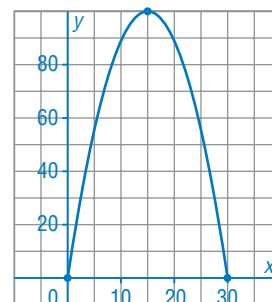**To determine the value of a, substitute the coordinates of an x-intercept: (30, 0)**
Substitute: $x = 30$, $y = 0$
$$0 = a(30 - 15)^2 + 100$$
$$-100 = 225a$$
$$a = -\frac{4}{9}$$
So, an equation that models this situation is: $y = -\frac{4}{9}(x - 15)^2 + 100$

C

12. Determine an equation of a quadratic function with x-intercepts of -2 and 4, that passes through the point E(2, −16).

An equation has the form $y = a(x - p)^2 + q$.
The axis of symmetry is midway between $x = -2$ and $x = 4$, that is $x = 1$. So, $p = 1$.
The equation becomes $y = a(x - 1)^2 + q$.
Substitute the coordinates of E(2, −16) and the coordinates of an x-intercept: $(-2, 0)$.

Substitute: **Substitute:**
$x = 2$, $y = -16$ **$x = -2$, $y = 0$**
$-16 = a(2 - 1)^2 + q$ **$0 = a(-2 - 1)^2 + q$**
$-16 = a + q$ ①   **$0 = 9a + q$ ②**
From ②, $q = -9a$
Substitute $q = -9a$ in ①.
$-16 = a - 9a$
$-16 = -8a$
$a = 2$
Substitute for a in ②.
$0 = 9(2) + q$
$-18 = q$
So, the equation of the function is: $y = 2(x - 1)^2 - 18$