Lesson 4.6 Exercises, pages 306–311

A

3. Identify the x-intercepts of the graph of each quadratic function.

a) $y = (x + 3)(x - 2)$

The x-intercepts are: -3 and 2

b) $y = (x - 4)(x - 5)$

The x-intercepts are: 4 and 5

c) $y = (x + 1)(x + 7)$

The x-intercepts are: -7 and -1

d) $y = -x(x + 5)$

The x-intercepts are: -5 and 0

e) $y = -(2x + 5)(3x - 3)$

The x-intercepts are: $-\frac{5}{2}$ and 1

f) $y = (4x - 1)(5x + 1)$

The x-intercepts are: $-\frac{1}{5}$ and $\frac{1}{4}$

4. Determine the zeros of each quadratic function.

a) $y = x^2 + 7x + 12$

$y = (x + 3)(x + 4)$

The zeros are: -4 and -3

b) $y = 2x^2 - 13x - 7$

$$\begin{aligned} y &= 2x^2 - 14x + x - 7 \\ &= 2x(x - 7) + (x - 7) \\ &= (2x + 1)(x - 7) \end{aligned}$$

The zeros are: $-\frac{1}{2}$ and 7

B

5. Determine the intercepts, the equation of the axis of symmetry, and the coordinates of the vertex of the graph of each quadratic function.

a) $y = 2x^2 - 20x + 32$

The y-intercept is 32.
Factor the equation.
$y = 2x^2 - 20x + 32$
$= 2(x^2 - 10x + 16)$
$= 2(x - 8)(x - 2)$
The x-intercepts are: 2 and 8
The mean of the intercepts is:
$\frac{2 + 8}{2} = 5$
So, the equation of the axis of symmetry is: $x = 5$
Substitute $x = 5$ in
$y = 2x^2 - 20x + 32$
$= 2(5)^2 - 20(5) + 32$
$= -18$
The coordinates of the vertex are: $(5, -18)$

b) $y = -\frac{1}{3}x^2 + 3x - 6$

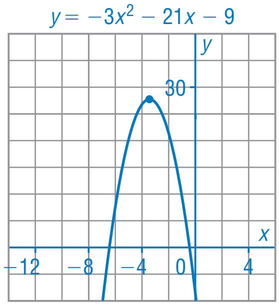
The y-intercept is -6.
Factor the equation.
$y = -\frac{1}{3}x^2 + 3x - 6$
$= -\frac{1}{3}(x^2 - 9x + 18)$
$= -\frac{1}{3}(x - 3)(x - 6)$
The x-intercepts are: 3 and 6
The mean is: $\frac{3 + 6}{2} = 4.5$
So, the equation of the axis of symmetry is: $x = 4.5$
Substitute $x = 4.5$ in
$y = -\frac{1}{3}x^2 + 3x - 6$
$y = -\frac{1}{3}(4.5)^2 + 3(4.5) - 6$
$= 0.75$
The coordinates of the vertex are: $(4.5, 0.75)$

6. Sketch a graph of each quadratic function. List the characteristics you used.

a) $y = -3x^2 - 21x - 9$

Discriminant is: $(-21)^2 - 4(-3)(-9) = 333$
Complete the square.
$y = -3x^2 - 21x - 9$
$y = -3\left(x^2 + 7x + \frac{49}{4} - \frac{49}{4}\right) - 9$
$y = -3(x + 3.5)^2 + 27.75$
The graph of $y = -3x^2 - 21x - 9$ opens down and is congruent to the graph of $y = -3x^2$. On a grid, mark a point at the vertex: $(-3.5, 27.75)$. Use the step pattern. Multiply each vertical step by -3.

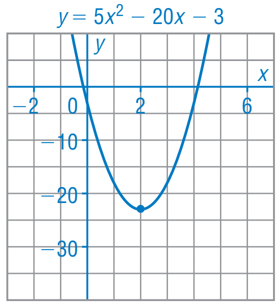
$y = -3x^2 - 21x - 9$



b) $y = 5x^2 - 20x - 3$

Discriminant is: $(-20)^2 - 4(5)(-3) = 460$
Complete the square.
$y = 5x^2 - 20x - 3$
$y = 5(x^2 - 4x + 4 - 4) - 3$
$y = 5(x - 2)^2 - 23$
The graph of $y = 5x^2 - 20x - 3$ opens up and is congruent to the graph of $y = 5x^2$. On a grid, mark a point at the vertex: $(2, -23)$. Use the step pattern. Multiply each vertical step by 5.

$y = 5x^2 - 20x - 3$



7. Determine the coordinates of the vertex of the graph of $y = -5x^2 + 2.5x + 7.5$ by factoring and by completing the square. Which strategy do you prefer? Why?

Completing the square:

$$\begin{aligned} y &= -5x^2 + 2.5x + 7.5 \\ &= -5(x^2 - 0.5x) + 7.5 \\ &= -5(x^2 - 0.5x + 0.0625 - 0.0625) + 7.5 \\ &= -5(x^2 - 0.5x + 0.0625) - 5(-0.0625) + 7.5 \\ &= -5(x - 0.25)^2 + 7.8125 \end{aligned}$$

The coordinates of the vertex are: (0.25, 7.8125)

Factoring:

$$\begin{aligned} y &= -5x^2 + 2.5x + 7.5 \\ &= -5(x^2 - 0.5x - 1.5) \\ &= -5(x - 1.5)(x + 1.0) \end{aligned}$$

The x-intercepts are: 1.5 and -1. The mean of the intercepts is:

$$\frac{-1 + 1.5}{2} = 0.25$$

So, the equation of the axis of symmetry is: $x = 0.25$

Substitute $x = 0.25$ in $y = -5x^2 + 2.5x + 7.5$

$$\begin{aligned} &= -5(0.25)^2 + 2.5(0.25) + 7.5 \\ &= 7.8125 \end{aligned}$$

The coordinates of the vertex are: (0.25, 7.8125)

I prefer the completing the square strategy because I can find the coordinates of the vertex in one step.

8. a) The graph of a quadratic function passes through A(1, 8) and has x-intercepts 2 and 5. Write an equation of the graph in factored form.

Use $y = a(x - x_1)(x - x_2)$ Substitute: $x_1 = 2$ and $x_2 = 5$

$y = a(x - 2)(x - 5)$ Substitute for A(1, 8).

$8 = a(1 - 2)(1 - 5)$

$8 = 4a$

$a = 2$

In factored form, the equation is: $y = 2(x - 2)(x - 5)$

b) The graph of a quadratic function passes through B(1, −7), and the zeros of the function are −6 and −1. Write an equation of the graph in general form.

Use $y = a(x - x_1)(x - x_2)$ Substitute: $x_1 = -6$ and $x_2 = -1$

$y = a(x + 6)(x + 1)$ Substitute for B(1, −7).

$-7 = a(1 + 6)(1 + 1)$

$-7 = 14a$

$a = -0.5$

In factored form, the equation is: $y = -0.5(x + 6)(x + 1)$

Expand to write the equation in general form.

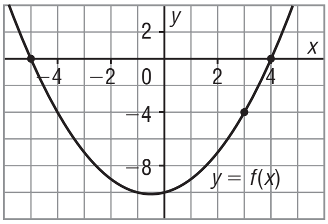
$y = -0.5(x + 6)(x + 1)$

$y = -0.5(x^2 + 7x + 6)$

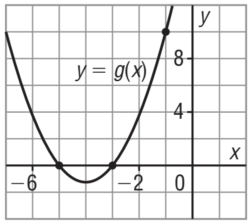
$y = -0.5x^2 - 3.5x - 3$

9. For each graph of a quadratic function, write the equation in the form given.

a) factored form



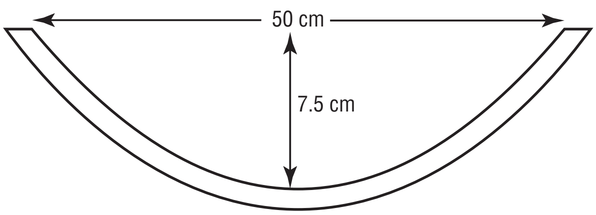
b) general form



x-intercepts: $-5, 4$
passes through $(3, -4)$
Use $y = a(x - x_1)(x - x_2)$
Substitute: $x_1 = -5$ and $x_2 = 4$
$y = a(x + 5)(x - 4)$
Substitute for $(3, -4)$.
$-4 = a(3 + 5)(3 - 4)$
$-4 = -8a$
$a = 0.5$
In factored form, the equation is:
$y = 0.5(x + 5)(x - 4)$

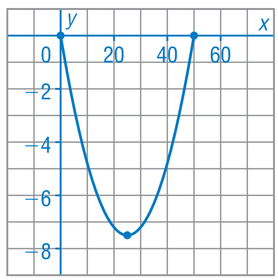
x-intercepts: $-3, -5$
passes through $(-1, 10)$
Use $y = a(x - x_1)(x - x_2)$
Substitute: $x_1 = -3$ and $x_2 = -5$
$y = a(x + 3)(x + 5)$
Substitute for $(-1, 10)$.
$10 = a(-1 + 3)(-1 + 5)$
$10 = 8a$
$a = 1.25$
In factored form, the equation is:
$y = 1.25(x + 3)(x + 5)$
Expand to write in general form:
$y = 1.25(x^2 + 8x + 15)$
$y = 1.25x^2 + 10x + 18.75$

10. The cross section of a satellite dish is parabolic. The parabola has a maximum depth of 7.5 cm and a width of 50 cm.



a) Determine an equation to model the parabolic dish.

Sample response: Place one end of the dish at the origin. Since the dish is 50 cm wide, the other end of the dish has coordinates $(50, 0)$.
The maximum depth of the dish is 7.5 cm, so the vertex has coordinates $(25, -7.5)$.

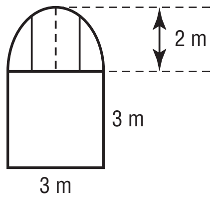


Use the factored form: $y = a(x - x_1)(x - x_2)$
Substitute: $x_1 = 0$ and $x_2 = 50$
The equation becomes: $y = ax(x - 50)$
Substitute: $x = 25$ and $y = -7.5$
$-7.5 = a(25)(25 - 50)$
$-7.5 = -625a$
$a = 0.012$
So, an equation that models the parabolic dish is: $y = 0.012x(x - 50)$

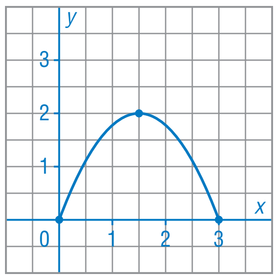
b) How deep is the dish 10 cm from its edge?

Use the equation: $y = 0.012x(x - 50)$
Substitute: $x = 10$
$y = 0.012(10)(10 - 50)$
$y = -4.8$
Since the point $(10, -4.8)$ is 4.8 units below the x-axis, the dish has a depth of 4.8 cm.

11. The top of a window is a parabolic arch. The dimensions are shown on the diagram. Additional supports are to be added halfway between each edge and the centre of the window. What is the length of each support?



Place one end of the arch at the origin.
Since the window is 3 m wide, the other end of the arch has coordinates (3, 0).
The maximum height of the arch is 2 m, so the vertex has coordinates (1.5, 2).
Use the factored form: $y = a(x - x_1)(x - x_2)$
Substitute: $x_1 = 0$ and $x_2 = 3$
The equation becomes: $y = ax(x - 3)$
Substitute: $x = 1.5$ and $y = 2$
$2 = a(1.5)(1.5 - 3)$
$2 = -2.25a$
$a = -\frac{8}{9}$
So, an equation that models the parabolic arch is: $y = -\frac{8}{9}x(x - 3)$



The centre of the window is the axis of symmetry: $x = 1.5$
So, the vertical line halfway between the centre of the window and the origin is: $x = \frac{1.5}{2}$, or 0.75
Substitute $x = 0.75$.
$y = -\frac{8}{9}(0.75)(0.75 - 3)$
$y = 1.5$
The two supports are congruent. Each support has length 1.5 m.

12. A student says that the quadratic function $y + 15 = (x + 2)(x + 4)$ has zeros -2 and -4. Explain the student's error and determine the zeros.

The equation is not written in factored form: $y = a(x - x_1)(x - x_2)$
$y + 15 = (x + 2)(x + 4)$
$y = (x + 2)(x + 4) - 15$
$y = x^2 + 6x - 7$
$y = (x + 7)(x - 1)$
So, the zeros of the equation are -7 and 1.

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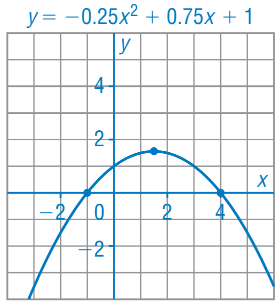
13. Sketch a graph of this quadratic function.

$y = -0.25x^2 + 0.75x + 1$

Discriminant is:
$(0.75)^2 - 4(-0.25)(1) = 1.5625$
Since 1.5625 is a perfect square, the equation factors.
$y = -0.25x^2 + 0.75x + 1$
$y = -0.25(x^2 - 3x - 4)$
$y = -0.25(x - 4)(x + 1)$
The x-intercepts are: -1 and 4. The mean of the intercepts is:
$\frac{-1 + 4}{2} = 1.5$
So, the equation of the axis of symmetry is: $x = 1.5$
Substitute $x = 1.5$ in $y = -0.25x^2 + 0.75x + 1$
$= -0.25(1.5)^2 + 0.75(1.5) + 1$
$= 1.5625$
The coordinates of the vertex are: (1.5, 1.5625)
The graph of $y = -0.25x^2 + 0.75x + 1$ is congruent to the graph of $y = -0.25x^2$.



C

14. The factored form of the equation of a quadratic function is $y = a(x - x_1)(x - x_2)$. The vertex form is $y = a(x - p)^2 + q$. Express p and q in terms of x_1 and x_2.

From the factored form of the equation, $y = a(x - x_1)(x - x_2)$, the x-intercepts of the graph of the equation are x_1 and x_2. The mean of the x-intercepts is $\frac{x_1 + x_2}{2}$, so the equation of the axis of symmetry is $x = \frac{x_1 + x_2}{2}$. So, the x-coordinate of the vertex is: $\frac{x_1 + x_2}{2}$.

To determine the y-coordinate of the vertex, substitute $x = \frac{x_1 + x_2}{2}$ in the factored form of the equation.

$y = a(x - x_1)(x - x_2)$

$y = a\left(\frac{x_1 + x_2}{2} - x_1\right)\left(\frac{x_1 + x_2}{2} - x_2\right)$

$y = a\left(\frac{x_1 + x_2 - 2x_1}{2}\right)\left(\frac{x_1 + x_2 - 2x_2}{2}\right)$

$y = a\left(\frac{-x_1 + x_2}{2}\right)\left(\frac{x_1 - x_2}{2}\right)$

$y = -a\left(\frac{x_1 - x_2}{2}\right)\left(\frac{x_1 - x_2}{2}\right)$

$y = -\frac{a}{4}(x_1 - x_2)^2$

From the vertex form of the equation, $y = a(x - p)^2 + q$, the coordinates of the vertex are (p, q). Therefore, $p = \frac{x_1 + x_2}{2}$ and $q = -\frac{a}{4}(x_1 - x_2)^2$.