PRACTICE TEST, pages 336–338

1. Multiple Choice Which set of data best describes the graph of the quadratic function $y = -4(x + 3)^2 - 7$?

A. Vertex: $(3, -7)$; opens down; congruent to $y = 4x^2$

B. Vertex: $(-3, -7)$; opens up; congruent to $y = 4x^2$

C. Vertex: $(-3, 7)$; opens up; congruent to $y = 4x^2$

(D.) Vertex: $(-3, -7)$; opens down; congruent to $y = 4x^2$

2. Multiple Choice Which equations represent quadratic functions?

I. $y = 2(x - 1)(x + 2)$ **II.** $y = -3(x + 1)^2 + 1$

III. $y = \frac{1}{x^2 - 1}$ **IV.** $y = 0.5x^2 - 1$

A. II only **B.** I and II **(C.)** I, II, and IV **D.** All of them

3. For the quadratic function below, sketch a graph, and identify:

a) the intercepts **b)** the coordinates of the vertex

c) the equation of the axis of symmetry

d) the domain of the function **e)** the range of the function

$y = 4x^2 + 8x - 60$

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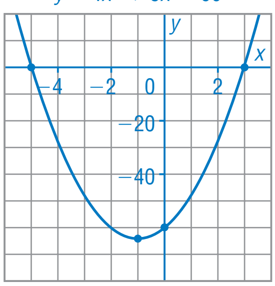


a) y-intercept: -60

x-intercepts:

$0 = 4x^2 + 8x - 60$

$0 = 4(x^2 + 2x - 15)$

$0 = 4(x + 5)(x - 3)$

So, $x = -5$ and $x = 3$

x-intercepts: $-5, 3$

b) The mean of the intercepts is:

$\frac{-5 + 3}{2} = -1$

Substitute $x = -1$ in $y = 4x^2 + 8x - 60$

$= 4(-1)^2 + 8(-1) - 60$

$= -64$

The coordinates of the vertex are: $(-1, -64)$

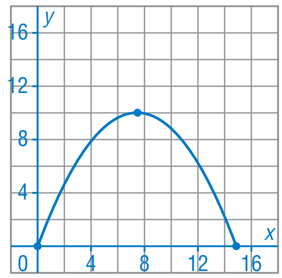
c) The equation of the axis of symmetry is: $x = -1$

d) Domain: $x \in \mathbb{R}$

e) The graph opens up, so the vertex is a minimum point with y-coordinate -64. The range is: $y \geq -64, y \in \mathbb{R}$

4. A basketball is thrown into the air from ground level and its path is a parabola. It reaches a maximum height of 10 m and lands 15 m from where it was thrown.

a) Determine an equation that models the path of the ball.



Sample response: Sketch the parabola on the coordinate plane, with the x-axis representing the ground. Assume the origin represents the point from which the basketball is thrown.
The ball lands 15 m from where it was thrown, so an x-intercept is 15.
The axis of symmetry is midway between $x = 0$ and $x = 15$, or $x = \frac{15}{2}$.
The maximum height is 10 m, so the vertex is at $\left(\frac{15}{2}, 10\right)$.

So, an equation has the form $y = a\left(x - \frac{15}{2}\right)^2 + 10$.
To determine the value of a, substitute the coordinates of an x-intercept: (15, 0)

$$0 = a\left(15 - \frac{15}{2}\right)^2 + 10$$
$$-10 = \frac{225}{4}a$$
$$a = -\frac{40}{225}, \text{ or } -\frac{8}{45}$$

So, an equation that models the path of the ball is:

$$y = -\frac{8}{45}\left(x - \frac{15}{2}\right)^2 + 10$$

b) What is the height of the ball at a point 3 m beyond where it was thrown, measured horizontally? How far is the ball from where it was thrown when its height has this value again? What assumptions did you make?

Substitute: $x = 3$

$$y = -\frac{8}{45}\left(3 - \frac{15}{2}\right)^2 + 10$$
$$y = 6.4$$

The height of the ball is 6.4 m. Since the graph is symmetrical about the axis of symmetry, $x = \frac{15}{2}$, the ball will have a height of 6.4 m at a point 3 m measured horizontally from where the ball lands; that is, 15 m − 3 m = 12 m from the origin. I assumed the ball was not touched by another player and that wind did not interfere with the path of the ball.

5. The sum of two numbers is 36. Their product is a maximum. Determine the numbers.

Let one number be represented by x. Then the other number is: $36 - x$
The product, P, of the numbers is: $x(36 - x)$
An equation is: $P = x(36 - x)$, or $P = -x^2 + 36x$
The coefficient of x^2 is negative, so the graph has a maximum value.
From the equation $P = x(36 - x)$, the x-intercepts are: 0, 36
So, the x-coordinate of the vertex is 18. So, one number is 18.
The other number is: $36 - 18 = 18$
The numbers are 18 and 18.

6. A store sells a calculator for \$8.00. At that price, the store sells approximately 1000 calculators per month. The store manager estimates that for every \$0.50 decrease in price, she will sell 200 more calculators. What is the price of a calculator that will maximize the revenue?

Let x represent the number of \$0.50 decreases in the price of a calculator. When the cost is \$8, 1000 are sold for a revenue of: \$8(1000) = \$8000. When the cost is \$(8 − 0.5x), (1000 + 200x) are sold for a revenue of \$(8 − 0.5x)(1000 + 200x).
Let the revenue be R dollars.
An equation is: $R = (8 - 0.5x)(1000 + 200x)$
Use a graphing calculator to graph the equation.
From the graph, the maximum revenue is about \$11 025 when the number of \$0.50 decreases is 5.5.
The number of decreases is a whole number, so round 5.5 to 5 or 6.
Five decreases of \$0.50 mean that the calculator will now cost:
\$8 − 5(\$0.50) = \$5.50
Six decreases of \$0.50 mean that the calculator will now cost:
\$8 − 6(\$0.50) = \$5.00
To maximize the revenue, the calculator should sell for \$5.50 or \$5.