Lesson 5.3 Exercises, pages 374–380

A

4. Determine whether each ordered pair is a solution of the quadratic inequality: $y \geq 3x^2 - 4$

a) $(-3, 8)$

Substitute each ordered pair in $y \geq 3x^2 - 4$.

L.S. = 8; R.S.: $3(-3)^2 - 4 = 23$
Since $8 < 23$, the ordered pair is not a solution.

b) $(0, 5)$

L.S. = 5; R.S.: $3(0)^2 - 4 = -4$
Since $5 > -4$, the ordered pair is a solution.

c) $(4, 44)$

L.S. = 44; R.S.: $3(4)^2 - 4 = 44$
Since $44 = 44$, the ordered pair is a solution.

d) $(-5, 12)$

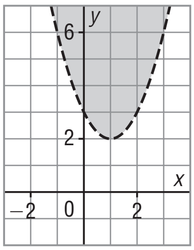
L.S. = 12; R.S.: $3(-5)^2 - 4 = 71$
Since $12 < 71$, the ordered pair is not a solution.

5. Match each inequality with a graph below.

i) $y < (x + 1)^2 - 2$ **ii)** $y \geq (x - 2)^2 - 1$

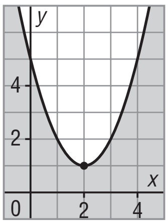
iii) $y > (x - 1)^2 + 2$ **iv)** $y \leq (x - 2)^2 + 1$

a)



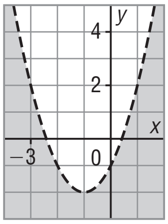
The parabola is congruent to $y = x^2$, and its vertex is (1, 2).
Its equation is:
$y = (x - 1)^2 + 2$
The inequality is:
$y > (x - 1)^2 + 2$

b)



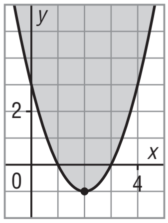
The parabola is congruent to $y = x^2$, and its vertex is (2, 1).
Its equation is:
$y = (x - 2)^2 + 1$
The inequality is:
$y \leq (x - 2)^2 + 1$

c)



The parabola is congruent to $y = x^2$, and its vertex is $(-1, -2)$.
Its equation is:
$y = (x + 1)^2 - 2$
The inequality is:
$y < (x + 1)^2 - 2$

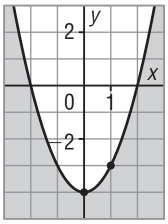
d)



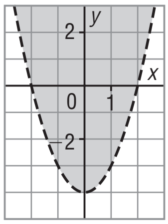
The parabola is congruent to $y = x^2$, and its vertex is $(2, -1)$.
Its equation is:
$y = (x - 2)^2 - 1$
The inequality is:
$y \geq (x - 2)^2 - 1$

6. Write an inequality to describe each graph.

a)



b)

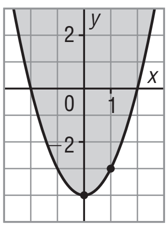


All 4 parabolas are congruent to $y = x^2$, and have vertex $(0, -4)$.
The equation is: $y = x^2 - 4$

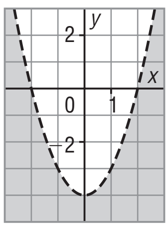
The curve is solid and the shaded region is below.
An inequality is: $y \leq x^2 - 4$

The curve is broken and the shaded region is above.
An inequality is: $y > x^2 - 4$

c)



d)



The curve is solid and the shaded region is above.
An inequality is: $y \geq x^2 - 4$

The curve is broken and the shaded region is below.
An inequality is: $y < x^2 - 4$

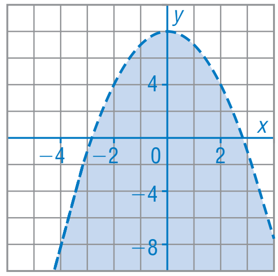
B

7. Graph each inequality. Write the coordinates of 3 points that satisfy the inequality.

a) $y < -x^2 + 8$

The graph of the related quadratic function is congruent to $y = -x^2$ and has vertex (0, 8).
The curve is broken and the region below is shaded.
3 points that satisfy the inequality have coordinates: (1, 4), (2, 1), (−1, 1)

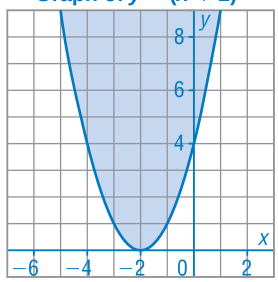
Graph of $y < -x^2 + 8$



b) $y \geq (x + 2)^2$

The graph of the related quadratic function is congruent to $y = x^2$ and has vertex (−2, 0).
The curve is solid and the region above is shaded.
3 points that satisfy the inequality have coordinates: (−2, 2), (−1, 3), (0, 5)

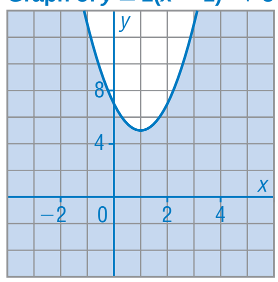
Graph of $y \geq (x + 2)^2$



c) $y \leq 2(x - 1)^2 + 5$

The graph of the related quadratic function is congruent to $y = 2x^2$ and has vertex (1, 5).
The curve is solid and the region below is shaded.
3 points that satisfy the inequality have coordinates: (−1, 4), (1, 2), (2, 4)

Graph of $y \leq 2(x - 1)^2 + 5$



8. Are the points on the graph of $y = 4x^2 - 3x + 7$ part of the solution of the inequality $y > 4x^2 - 3x + 7$? Explain your answer.

No, the points are not part of the solution, because the inequality indicates that y is greater than $4x^2 - 3x + 7$, and, on the line, $y = 4x^2 - 3x + 7$.

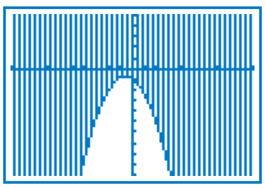
9. Use technology to graph each inequality. Sketch or print the graph.

a) $y \geq -\frac{2}{3}x^2 - \frac{4}{5}x - 1$

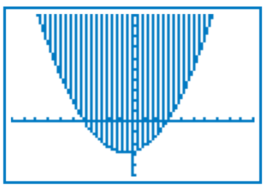
b) $y > 0.25x^2 + 0.3x - 2.8$

Graph the related quadratic functions.

a) The boundary is part of the graph.



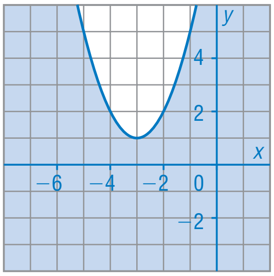
b) The boundary is not part of the graph.



10. Graph each inequality. Write the coordinates of 3 points that satisfy the inequality.

a) $y \leq x^2 + 6x + 10$

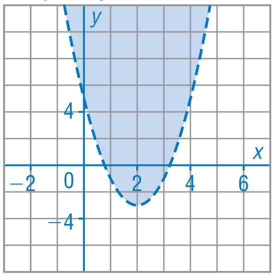
Graph of $y \leq x^2 + 6x + 10$



Complete the square for:
$y = x^2 + 6x + 10$
$y = x^2 + 6x + 9 - 9 + 10$
$y = (x + 3)^2 + 1$
The graph of this function is congruent to $y = x^2$ and has vertex $(-3, 1)$.
The curve is solid and the region below is shaded.
3 points that satisfy the inequality have coordinates: $(-4, 1)$, $(-2, 1)$, $(0, 3)$

b) $y > 2x^2 - 8x + 5$

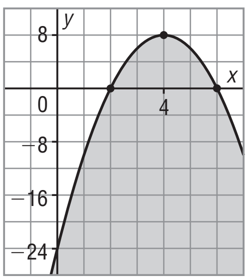
Graph of $y > 2x^2 - 8x + 5$



Complete the square for:
$y = 2x^2 - 8x + 5$
$y = 2(x^2 - 4x + 4 - 4) + 5$
$y = 2(x - 2)^2 - 3$
The graph of this function is congruent to $y = 2x^2$ and has vertex $(2, -3)$.
The curve is broken and the region above is shaded.
3 points that satisfy the inequality have coordinates: $(1, 4)$, $(2, 8)$, $(3, 1)$

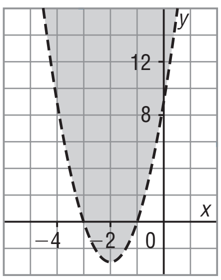
11. Write an inequality to describe each graph.

a)



The parabola is congruent to $y = -2x^2$, and has vertex $(4, 8)$. Its equation is:
$y = -2(x - 4)^2 + 8$
The curve is solid and the region below it is shaded.
An inequality is:
$y \leq -2(x - 4)^2 + 8$

b)



The parabola is congruent to $y = 3x^2$, and has vertex $(-2, -3)$. Its equation is:
$y = 3(x + 2)^2 - 3$
The curve is broken and the region above it is shaded.
An inequality is:
$y > 3(x + 2)^2 - 3$

12. **a)** For A$(-1, a)$ to be a solution of $y > -2x^2 + 5$, what must be true about a?

In $y > -2x^2 + 5$, substitute: $x = -1$, $y = a$
$a > -2(-1)^2 + 5$
$a > 3$

b) For B$(b, 6)$ to be a solution of $y > x^2 - 5$, what must be true about b?

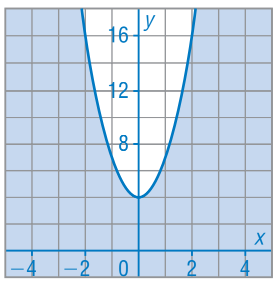
In $y > x^2 - 5$, substitute: $x = b$, $y = 6$
$6 > b^2 - 5$
$b^2 < 11$
$b < \sqrt{11}$ or $b > -\sqrt{11}$
That is, $-\sqrt{11} < b < \sqrt{11}$

13. Two numbers are related in this way: three times the square of one number is greater than or equal to the other number minus 4.

a) Graph an inequality that represents this relationship.

Let the numbers be represented by x and y. An inequality is:
$3x^2 \geq y - 4$, or $y \leq 3x^2 + 4$
The graph of the related function is congruent to $y = 3x^2$ and its vertex is (0, 4).
The curve is solid, with the region below it shaded.

Graph of $y \leq 3x^2 + 4$



b) Use the graph to list three pairs of integer values for the two numbers.

Three pairs of integer values are:
$(-2, 16)$, $(-1, -2)$, $(1, -10)$

14. Graph this quadratic inequality: $\frac{(x + 1)^2}{2} - 3 > \frac{y}{3}$

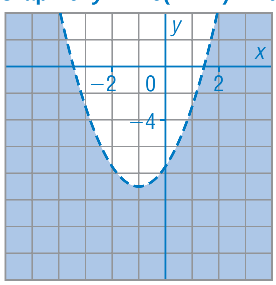
$\frac{(x + 1)^2}{2} - 3 > \frac{y}{3}$ Multiply by 3.

$\frac{3(x + 1)^2}{2} - 3(3) > y$

$y < 1.5(x + 1)^2 - 9$

The graph of the related function is congruent to $y = 1.5x^2$ and its vertex is $(-1, -9)$.
The curve is broken, with the region below it shaded.

Graph of $y < 1.5(x + 1)^2 - 9$



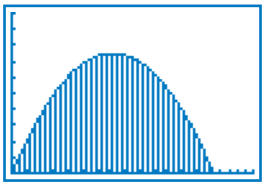
15. An arch that supports a bridge over a river is parabolic and spans a horizontal distance of 250 m. An equation of the parabola is $y = -0.0048x^2 + 1.2x$, where y metres is the height of a point on the arch above the river, and x metres is the horizontal distance to that point measured from one end of the arch.

a) Write an inequality to represent the cross-sectional region between the arch and the river.

Since both x and y are positive, the graph of the parabola is only in Quadrant 1. An inequality is: $y < -0.0048x^2 + 1.2x, x > 0, y > 0$

b) Graph the inequality, then sketch it below.

Use a graphing calculator. Graph $y = -0.0048x^2 + 1.2x, x > 0, y > 0$ in Quadrant 1 with a broken curve. Shade the region between the curve and the x-axis.



c) The tallest mast of a ship is 37 m above water level. Can the ship pass under the arch when it is 50 m from one end of the arch? Justify your answer.

Check whether the point (50, 37) satisfies the inequality.
In $y < -0.0048x^2 + 1.2x$, substitute: $x = 50, y = 37$
L.S. = 37 R.S.: $-0.0048(50)^2 + 1.2(50) = 48$
Since L.S. < R.S., the ship can pass under the arch.

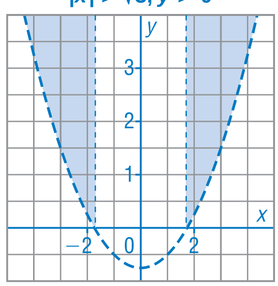
C

16. The length of a rectangle is 4 times a number. The width of the rectangle is 3 less than the square of another number. The length of the rectangle is greater than its width.

a) Sketch a graph to represent this situation.

Let the length of the rectangle be represented by $4y$ units, and the width by $(x^2 - 3)$ units. The length is greater than the width so an inequality is: $4y > x^2 - 3$, or $y > 0.25x^2 - 0.75$ The graph of the related function is congruent to $y = 0.25x^2$ and its vertex is $(0, -0.75)$. The curve is broken. Since y cannot be negative, only the region above the x-axis is shaded. Also, $x^2 - 3 > 0$, so $x > \sqrt{3}$ or $x < -\sqrt{3}$ Shade the region that satisfies these inequalities: $y > 0.25x^2 - 0.75$, $y > 0$, $|x| > \sqrt{3}$

Graph of $y > 0.25x^2 - 0.75$, $|x| > \sqrt{3}$, $y > 0$



b) Use the graph to list three possible sets of dimensions for the rectangle.

Three sets of coordinates are:
(−2, 1), (2, 2), (3, 3)
So, three possible sets of dimensions are:
width: $(-2)^2 - 3 = 1$; length: $4(1) = 4$
width: $2^2 - 3 = 1$; length: $4(2) = 8$
width: $3^2 - 3 = 6$; length: $4(3) = 12$
Possible dimensions are: 1 unit by 4 units; 1 unit by 8 units, 6 units by 12 units

17. Write a quadratic inequality in 2 variables that has these 3 points as solutions: A(0, −3), B(−3, 0), and C(3, 3)

Sample response: Visualize the points on a grid. The highest point is (3, 3). Visualize a parabola that lies above the points; for example, the parabola opens up, has vertex (4, 4), and is congruent to $y = x^2$. Its equation is: $y = (x - 4)^2 + 4$
So, an inequality is: $y < (x - 4)^2 + 4$