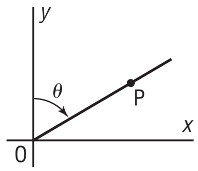
Lesson 6.1 Exercises, pages 431–438

A

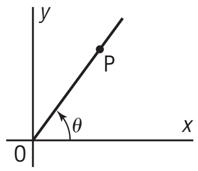
3. State whether each diagram represents an angle in standard position. Explain your thinking.

a)



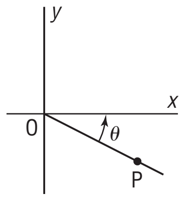
The angle is not in standard position because it is not measured from the *x*-axis.

b)



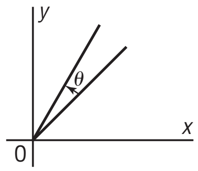
The angle is in standard position because it is measured counterclockwise from the positive *x*-axis.

c)



The angle is not in standard position because it is not measured from the *x*-axis.

d)



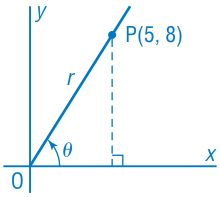
The angle is not in standard position because it is not measured from the *x*-axis.

B

4. Point P(5, 8) is on the terminal arm of an angle θ in standard position.

a) Sketch the angle.

Plot P(5, 8); draw a line through OP.
Label θ. Let the length of OP $= r$.



b) Determine the distance from the origin to P.

Use the Pythagorean Theorem in the right triangle formed by OP and the perpendicular from P to the x-axis.

$$r^2 = 5^2 + 8^2$$
$$r^2 = 89$$
$$r = \sqrt{89}$$

c) Write the primary trigonometric ratios of θ.

$x = 5$, $y = 8$, $r = \sqrt{89}$

$$\sin\theta = \frac{y}{r} = \frac{8}{\sqrt{89}} \qquad \cos\theta = \frac{x}{r} = \frac{5}{\sqrt{89}} \qquad \tan\theta = \frac{y}{x} = \frac{8}{5}$$

d) What is the measure of θ to the nearest degree?

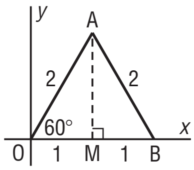
Use: $\tan\theta = \frac{8}{5}$

$$\theta = \tan^{-1}\left(\frac{8}{5}\right)$$
$$\theta = 57.9946...°$$

θ is approximately 58°.

5. a) Use this diagram to determine the exact primary trigonometric ratios of 60°.

You will need the results of question 5 in Lesson 6.2.



Use the Pythagorean Theorem in ΔOAM to determine the length of AM.
$AM^2 = 2^2 - 1^2$
$AM = \sqrt{3}$
The coordinates of A are $(1, \sqrt{3})$.
Then $x = 1$, $y = \sqrt{3}$, and $r = 2$

$\sin 60° = \frac{y}{r} = \frac{\sqrt{3}}{2}$ $\quad \cos 60° = \frac{x}{r} = \frac{1}{2}$ $\quad \tan 60° = \frac{y}{x} = \frac{\sqrt{3}}{1}$, or $\sqrt{3}$

b) Use the diagram in part a to determine the exact primary trigonometric ratios of 30°.

From the angle sum in ΔOAM, $\angle OAM = 30°$

$\sin 30° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2}$ $\quad \cos 30° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{\sqrt{3}}$

c) How are the values of the primary trigonometric ratios of 30° and 60° related? How can you predict the relationship by inspecting the triangles?

$\sin 30° = \cos 60°$; $\sin 60° = \cos 30°$; and $\tan 30° = \frac{1}{\tan 60°}$
The side that is adjacent to 60° is the side that is opposite 30°, and vice versa.

6. For each angle below, determine the exact coordinates of a point on the terminal arm of the angle in standard position.

a) 30°

Sample response: Choose a value for r, such as $r = 6$. Then:

$x = r \cos 30° = 6\left(\frac{\sqrt{3}}{2}\right)$, or $3\sqrt{3}$ $\quad y = r \sin 30° = 6\left(\frac{1}{2}\right)$, or 3

The exact coordinates are: $(3\sqrt{3}, 3)$

b) 45°

Sample response: Choose a value for r, such as $r = 6$. Then:

$x = r\cos 45°$ $\quad\quad$ $y = r\sin 45°$

$= 6\left(\frac{1}{\sqrt{2}}\right)$, or $\frac{6}{\sqrt{2}}$ $\quad\quad$ $= 6\left(\frac{1}{\sqrt{2}}\right)$, or $\frac{6}{\sqrt{2}}$

The exact coordinates are: $\left(\frac{6}{\sqrt{2}}, \frac{6}{\sqrt{2}}\right)$

c) 60°

Sample response: Choose a value for r, such as $r = 6$. Then:

$x = r\cos 60°$ $\quad\quad$ $y = r\sin 60°$

$= 6\left(\frac{1}{2}\right)$, or 3 $\quad\quad$ $= 6\left(\frac{\sqrt{3}}{2}\right)$, or $3\sqrt{3}$

The exact coordinates are: $(3, 3\sqrt{3})$

7. A support cable is anchored 15 m from the base of a pole and is attached to the pole 10 m above the ground.

a) Determine the length of the cable to the nearest tenth of a metre.

Sketch a diagram.
The length of the cable is OP.
Use the Pythagorean Theorem in ΔOPR.

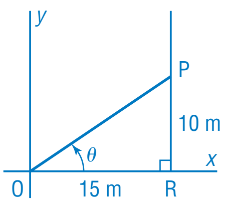
$OP^2 = 10^2 + 15^2$

$OP^2 = 325$

$OP = \sqrt{325}$

$OP = 18.0277...$

The cable is approximately 18.0 m long.



b) To the nearest degree, what angle does the cable make with the ground?

The angle is θ.

$\tan\theta = \frac{10}{15}$

$\theta = 33.6900...°$

The angle is approximately 34°.

8. a) Determine the distance of each point from the origin.

i) A(4, 6)

The distance is r.

$r^2 = 4^2 + 6^2$

$r^2 = 52$

$r = \sqrt{52}$

ii) B(7, 3)

The distance is r.

$r^2 = 7^2 + 3^2$

$r^2 = 58$

$r = \sqrt{58}$

b) Each point in part a is on the terminal arm of an angle θ in standard position. For each angle, determine $\cos\theta$, $\sin\theta$, $\tan\theta$, and the measure of θ to the nearest degree.

i) A(4, 6)

$\cos\theta = \frac{x}{r}$, or $\frac{4}{\sqrt{52}}$

$\sin\theta = \frac{y}{r}$, or $\frac{6}{\sqrt{52}}$

$\tan\theta = \frac{y}{x}$, or $\frac{6}{4} = 1.5$

Since $\tan\theta = 1.5$,

$\theta = 56.3099...°$

θ is approximately 56°.

ii) B(7, 3)

$\cos\theta = \frac{x}{r}$, or $\frac{7}{\sqrt{58}}$

$\sin\theta = \frac{y}{r}$, or $\frac{3}{\sqrt{58}}$

$\tan\theta = \frac{y}{x}$, or $\frac{3}{7}$

Since $\tan\theta = \frac{3}{7}$,

$\theta = 23.1985...°$

θ is approximately 23°.

9. Point P(x, y) is on the terminal arm of each angle below in standard position. The distance r between P and the origin is given. To the nearest tenth, determine the coordinates of P.

a) $20°$; $r = 10$

Use: $x = r\cos\theta$, $y = r\sin\theta$

Substitute: $r = 10$, $\theta = 20°$

$x = 10\cos 20°$

$= 9.3969...$

$y = 10\sin 20°$

$= 3.4202...$

The coordinates of P are approximately: (9.4, 3.4)

b) $80°$; $r = 5$

Use: $x = r\cos\theta$, $y = r\sin\theta$

Substitute: $r = 5$, $\theta = 80°$

$x = 5\cos 80°$

$= 0.8682...$

$y = 5\sin 80°$

$= 4.9240...$

The coordinates of P are approximately: (0.9, 4.9)

10. Each angle θ is in standard position in Quadrant 1.

a) $\cos\theta = \frac{5}{13}$; what are $\sin\theta$ and $\tan\theta$?

Use: $r^2 = x^2 + y^2$ Substitute: $r = 13$, $x = 5$

$13^2 = 5^2 + y^2$

$y^2 = 144$

$y = 12$

$\sin\theta = \frac{y}{r}$, or $\frac{12}{13}$; and $\tan\theta = \frac{y}{x}$, or $\frac{12}{5}$

b) $\sin\theta = \frac{2}{\sqrt{5}}$; what are $\cos\theta$ and $\tan\theta$?

Use: $r^2 = x^2 + y^2$ Substitute: $r = \sqrt{5}$, $y = 2$

$(\sqrt{5})^2 = x^2 + 2^2$

$x^2 = 1$

$x = 1$

$\cos\theta = \frac{x}{r}$, or $\frac{1}{\sqrt{5}}$; and $\tan\theta = \frac{y}{x}$, or $\frac{2}{1} = 2$

c) $\tan \theta = \frac{3}{\sqrt{7}}$; what are $\sin \theta$ and $\cos \theta$?

Use: $r^2 = x^2 + y^2$ Substitute: $x = \sqrt{7}$, $y = 3$

$r^2 = (\sqrt{7})^2 + 3^2$

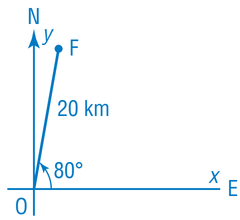
$r^2 = 16$

$r = 4$

$\sin \theta = \frac{y}{r}$, or $\frac{3}{4}$; and $\cos \theta = \frac{x}{r}$, or $\frac{\sqrt{7}}{4}$

11. A fire spotter sees smoke rising from a point that lies in a direction E80°N. He estimates that the distance from his location is about 20 km. The firefighters have to travel east then north to get to the fire. To the nearest kilometre, how far should the firefighters travel in each direction?

Sketch a diagram.



The distance due east is the x-coordinate of F.

$x = r \cos \theta$ Substitute: $r = 20$, $\theta = 80°$

$x = 20 \cos 80°$

$x = 3.4729\ldots$

The distance due north is the y-coordinate of F.

$y = r \sin \theta$ Substitute: $r = 20$, $\theta = 80°$

$y = 20 \sin 80°$

$y = 19.6961\ldots$

The firefighters should travel approximately 3 km east and 20 km north.

12. Determine the slope of the terminal arm for each angle in standard position. Give the answer to 1 decimal place.

a) 10° b) 50°

Slope is $\frac{\text{rise}}{\text{run}}$, which is $\frac{y}{x}$, where (x, y) are the coordinates of a point on the terminal arm of an angle θ.

$\frac{y}{x} = \tan \theta$

a) Slope is: $\tan 10° \doteq 0.2$ b) Slope is: $\tan 50° \doteq 1.2$

13. Use the trigonometric ratios for each of 30°, 45°, and 60° to verify that: $(\sin \theta)^2 + (\cos \theta)^2 = 1$

Substitute: $\theta = 30°$

L.S. $= (\sin 30°)^2 + (\cos 30°)^2$
$= 0.25 + 0.75$
$= 1$
$=$ R.S.

Substitute: $\theta = 45°$

L.S. $= (\sin 45°)^2 + (\cos 45°)^2$
$= 0.5 + 0.5$
$= 1$
$=$ R.S.

Substitute: $\theta = 60°$

L.S. $= (\sin 60°)^2 + (\cos 60°)^2$
$= 0.75 + 0.25$
$= 1$
$=$ R.S.

14. Explain why each of the following statements is true for $0° < \theta < 90°$.

a) $\cos (90° - \theta) = \sin \theta$

In a right triangle, when one acute angle is θ, then the other acute angle is $90° - \theta$.
The cosine of one acute angle is equal to the sine of the other acute angle because the side that is adjacent to one angle is opposite the other angle.

b) $\sin (90° - \theta) = \cos \theta$

In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle because the side that is opposite one angle is adjacent to the other angle.

c) $\tan (90° - \theta) = \frac{1}{\tan \theta}$

In a right triangle, the tangent of one angle is the reciprocal of the tangent of the other angle because the side that is adjacent to one angle is opposite the other angle, and vice versa.

C

15. Point P is on the terminal arm of an angle in standard position in Quadrant 1. The distance r between P and the origin is given. Determine possible coordinates for P.

a) $\sqrt{29}$

Sample response: The length $r = \sqrt{29}$ is the hypotenuse of a right triangle with legs x and y, which are the coordinates of point P. The square of the hypotenuse is 29. Use guess and test to find two numbers whose squares have a sum of 29.
$29 = 4 + 25$, or $2^2 + 5^2$
So, possible coordinates for P are (2, 5) or (5, 2).

b) $\sqrt{74}$

Sample response: Use guess and test to find two numbers whose squares have a sum of 74.
$74 = 25 + 49$, or $5^2 + 7^2$
So, possible coordinates for P are (5, 7) or (7, 5).

16. a) Use the trigonometric ratios for each of 30°, 45°, and 60° to verify that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Substitute: $\theta = 30°$

$$\text{L.S.} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\text{R.S.} = \frac{\sin 30°}{\cos 30°}$$

$$\text{R.S.} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Substitute: $\theta = 45°$

$$\text{L.S.} = \tan 45° = 1$$

$$\text{R.S.} = \frac{\sin 45°}{\cos 45°}$$

$$\text{R.S.} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\text{L.S.} = \tan 60° = \sqrt{3}$$

$$\text{R.S.} = \frac{\sin 60°}{\cos 60°}$$

$$\text{R.S.} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

b) Explain why $\tan\theta = \frac{\sin\theta}{\cos\theta}$ for $0° \leq \theta < 90°$.

What happens when $\theta = 90°$?

For an angle θ in standard position, with P(x, y) on its terminal arm and OP = r:

$$\tan\theta = \frac{y}{x} \qquad \frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}}, \text{ or } \frac{y}{x}$$

When $\theta = 90°$, $x = 0$, and $\tan\theta$ is undefined.