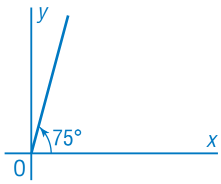
Lesson 6.2 Exercises, pages 448–458

A

3. Sketch each angle in standard position.

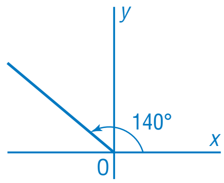
a) 75°

Since the angle is between 0° and 90°, the terminal arm is in Quadrant 1.



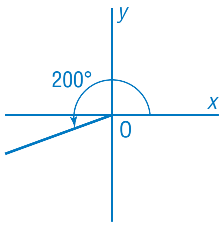
b) 140°

Since the angle is between 90° and 180°, the terminal arm is in Quadrant 2.



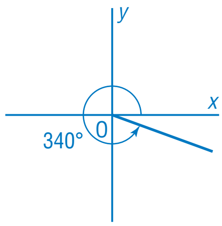
c) 200°

Since the angle is between 180° and 270°, the terminal arm is in Quadrant 3.



d) 340°

Since the angle is between 270° and 360°, the terminal arm is in Quadrant 4.



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4. a) Determine the reference angle for each angle in standard position.

i) 34°

The angle is in Quadrant 1, so its reference angle is 34°.

ii) 98°

The angle is in Quadrant 2, so its reference angle is:
180° − 98° = 82°

iii) 241°

The angle is in Quadrant 3, so its reference angle is:
241° − 180° = 61°

iv) 290°

The angle is in Quadrant 4, so its reference angle is:
360° − 290° = 70°

b) For each angle in part a, determine the other angles between 90° and 360° that have the same reference angle.

i) 34°

The angles with the same reference angle are:
180° − 34° = 146°
180° + 34° = 214°
360° − 34° = 326°

ii) 98°

The angles with the same reference angle are:
180° + 82° = 262°
360° − 82° = 278°

iii) 241°

The angles with the same reference angle are:
180° − 61° = 119°
360° − 61° = 299°

iv) 290°

The angles with the same reference angle are:
180° − 70° = 110°
180° + 70° = 250°

5. Determine the quadrant in which the terminal arm of each angle in standard position lies.

a) 280°

Since the angle is between 270° and 360°, the terminal arm lies in Quadrant 4.

b) 88°

Since the angle is between 0° and 90°, the terminal arm lies in Quadrant 1.

c) 191°

Since the angle is between 180° and 270°, the terminal arm lies in Quadrant 3.

d) 103°

Since the angle is between 90° and 180°, the terminal arm lies in Quadrant 2.

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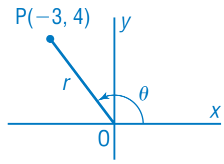
B

6. Point P(−3, 4) is a terminal point of an angle θ in standard position.

a) Sketch the angle.

Plot P(−3, 4); draw a line through OP. Label θ. Let the length of OP = r.



b) What is the distance from the origin to P?

Use: $r = \sqrt{x^2 + y^2}$
Substitute: $x = -3$, $y = 4$
$r = \sqrt{(-3)^2 + 4^2}$
$r = \sqrt{25}$
$r = 5$

c) Write the primary trigonometric ratios of θ.

$x = -3$, $y = 4$, $r = 5$

$\sin\theta = \frac{y}{r} = \frac{4}{5}$

$\cos\theta = \frac{x}{r} = \frac{-3}{5}$, or $-\frac{3}{5}$

$\tan\theta = \frac{y}{x} = \frac{4}{-3}$, or $-\frac{4}{3}$

d) To the nearest degree, what is θ?

Use: $\sin\theta = \frac{4}{5}$
The reference angle is:
$\sin^{-1}\left(\frac{4}{5}\right) = 53.1301\ldots°$

θ is approximately 180° − 53°, or 127°.

7. Each angle θ is in standard position. State the quadrants in which the terminal arm of the angle could lie.

a) $\cos\theta = -\frac{2}{3}$

$\cos\theta = \frac{x}{r}$, so x is negative; and the terminal arm lies in Quadrant 2 or 3.

b) $\tan\theta = -4$

$\tan\theta = \frac{y}{x}$, so if x is negative, the terminal arm lies in Quadrant 2; if y is negative, the terminal arm lies in Quadrant 4.

c) $\sin\theta = \frac{1}{2}$

$\sin\theta = \frac{y}{r}$, so y is positive; and the terminal arm lies in Quadrant 1 or 2.

d) $\tan\theta = 0.25$

$\tan\theta = \frac{y}{x}$, so if both x and y are positive, the terminal arm lies in Quadrant 1; if both x and y are negative, the terminal arm lies in Quadrant 3.

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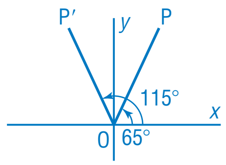
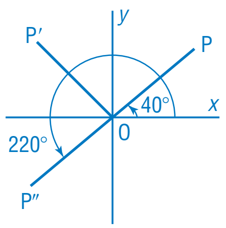
8. a) Choose an angle between 90° and 180°. Sketch a diagram to show that the angle can be formed by reflecting its reference angle in an axis or axes.

Sample response:
For 115°, its reference angle is:
180° − 115° = 65°
Draw 65° in Quadrant 1.
Choose a terminal point, P.
Reflect OP in the *y*-axis.
Then OP′ is the terminal arm of 115°.



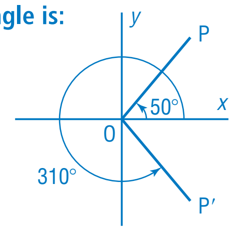
b) Repeat part a for an angle between 180° and 270°.

Sample response: For 220°, its reference angle is:
220° − 180° = 40°
Draw 40° in Quadrant 1.
Choose a terminal point, P.
Reflect OP in the *y*-axis, then reflect OP′ in the *x*-axis.
Then OP″ is the terminal arm of 220°.



c) Repeat part a for an angle between 270° and 360°.

Sample response: For 310°, its reference angle is:
360° − 310° = 50°
Draw 50° in Quadrant 1.
Choose a terminal point, P.
Reflect OP in the *x*-axis.
Then OP′ is the terminal arm of 310°.



9. Determine possible coordinates of a terminal point for each angle in standard position.

a) 315°

Sample response: Choose a value for r, such as $r = 4$. Then:

$x = r\cos 315°$ $\quad$ $y = r\sin 315°$

$= 4\left(\frac{1}{\sqrt{2}}\right)$, or $\frac{4}{\sqrt{2}}$ $\quad$ $= 4\left(-\frac{1}{\sqrt{2}}\right)$, or $-\frac{4}{\sqrt{2}}$

Possible coordinates are: $\left(\frac{4}{\sqrt{2}}, -\frac{4}{\sqrt{2}}\right)$

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b) 210°

Sample response: Choose a value for r, such as $r = 5$. Then:

$x = r \cos 210°$ $\quad$ $y = r \sin 210°$

$= 5\left(-\frac{\sqrt{3}}{2}\right)$, or $-\frac{5\sqrt{3}}{2}$ $\quad$ $= 5\left(-\frac{1}{2}\right)$, or $-\frac{5}{2}$

Possible coordinates are: $\left(-\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$

c) 120°

Sample response: Choose a value for r, such as $r = 6$. Then:

$x = r \cos 120°$ $\quad$ $y = r \sin 120°$

$= 6\left(-\frac{1}{2}\right)$, or -3 $\quad$ $= 6\left(\frac{\sqrt{3}}{2}\right)$, or $3\sqrt{3}$

Possible coordinates are: $(-3, 3\sqrt{3})$

10. Each point below lies on the terminal arm of an angle θ in standard position. Determine:

i) $\cos \theta$ **ii)** $\sin \theta$ **iii)** $\tan \theta$

iv) the value of θ to the nearest degree

a) A(−3, −4)

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-3)^2 + (-4)^2}$

$r = 5$

i) $\cos \theta = \frac{x}{r}$

$= \frac{-3}{5}$

ii) $\sin \theta = \frac{y}{r}$

$= \frac{-4}{5}$

iii) $\tan \theta = \frac{y}{x}$

$= \frac{-4}{-3}$, or $\frac{4}{3}$

iv) Use: $\tan \theta = \frac{4}{3}$

The reference angle is:

$\tan^{-1}\left(\frac{4}{3}\right) = 53.1301...°$

Since both x and y are negative, the terminal arm lies in Quadrant 3, and θ is approximately:

$180° + 53° = 233°$

b) B(−6, 0)

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-6)^2 + (0)^2}$

$r = 6$

i) $\cos \theta = \frac{x}{r}$

$= \frac{-6}{6}$

$= -1$

ii) $\sin \theta = \frac{y}{r}$

$= \frac{0}{6}$

$= 0$

iii) $\tan \theta = \frac{y}{x}$

$= \frac{0}{-6}$, or 0

iv) Since the terminal point lies on the negative x-axis, $\theta = 180°$

c) C(0, 2)

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(0)^2 + (2)^2}$

$r = 2$

i) $\cos\theta = \frac{x}{r}$
$= \frac{0}{2}$
$= 0$

ii) $\sin\theta = \frac{y}{r}$
$= \frac{2}{2}$
$= 1$

iii) $\tan\theta = \frac{y}{x}$
$= \frac{2}{0}$

$\tan\theta$ is undefined.

iv) Since the terminal point is on the positive y-axis, $\theta = 90°$

d) D(2, −1)

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(2)^2 + (-1)^2}$

$r = \sqrt{5}$

i) $\cos\theta = \frac{x}{r}$
$= \frac{2}{\sqrt{5}}$

ii) $\sin\theta = \frac{y}{r}$
$= \frac{-1}{\sqrt{5}}$

iii) $\tan\theta = \frac{y}{x}$
$= \frac{-1}{2}$, or $-\frac{1}{2}$

iv) Use: $\cos\theta = \frac{2}{\sqrt{5}}$

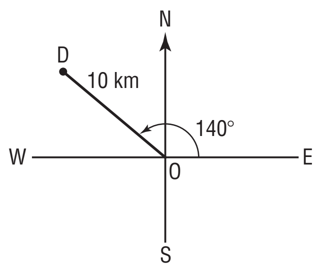
The reference angle is:

$\cos^{-1}\left(\frac{2}{\sqrt{5}}\right) = 26.5650...°$

Since x is positive and y is negative, the terminal arm lies in Quadrant 4 and θ is approximately:

$360° - 27° = 333°$

11. A hiker sketches a map of her destination, D, from her starting point, O.
The hiker can travel only west, then north.
To the nearest kilometre, how far must she hike to get to her destination?



The distance the hiker travels west is the absolute value of the x-coordinate of D.

$x = r\cos\theta$ Substitute: $r = 10$ and $\theta = 140°$

$x = 10\cos 140°$

$x = -7.6604...$

The distance the hiker travels north is the y-coordinate of D.

$y = r\sin\theta$ Substitute: $r = 10$ and $\theta = 140°$

$y = 10\sin 140°$

$y = 6.4278...$

In kilometres, the hiker travels: $7.6604... + 6.4278... = 14.0883...$

The hiker travels approximately 14 km.

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12. Explain why $(\sin\theta)^2 + (\cos\theta)^2 = 1$ for any angle θ in standard position in Quadrant 2.

In Quadrant 2, $\cos\theta = -\frac{x}{r}$ and $\sin\theta = \frac{y}{r}$

So, $(\sin\theta)^2 + (\cos\theta)^2 = \left(\frac{y}{r}\right)^2 + \left(-\frac{x}{r}\right)^2$

$$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$$

$$= \frac{y^2 + x^2}{r^2}$$

Since $y^2 + x^2 = r^2$,

$(\sin\theta)^2 + (\cos\theta)^2 = \frac{r^2}{r^2}$

$= 1$

13. The location of the terminal arm of an angle α and a trigonometric ratio of its reference angle θ are given.

a) The terminal arm lies in Quadrant 3, $\cos\theta = \frac{1}{4}$; what is $\tan\alpha$?

Let Q(x, y) be a point on the terminal arm of θ that is r units from O.

$\cos\theta = \frac{1}{4}$ and $\cos\theta = \frac{x}{r}$, so write $x = 1$ and $r = 4$

Determine the value of y.

$r^2 = x^2 + y^2$

$4^2 = 1^2 + y^2$

$y = \sqrt{15}$

Then P(-1, $-\sqrt{15}$) lies on the terminal arm of α.

$\tan\alpha = \frac{y}{x}$, so $\tan\alpha = \frac{-\sqrt{15}}{-1}$, or $\sqrt{15}$

b) The terminal arm lies in Quadrant 2, $\tan\theta = \frac{3}{7}$; what is $\sin\alpha$?

Consider point Q from part a.

$\tan\theta = \frac{3}{7}$ and $\tan\theta = \frac{y}{x}$, so $y = 3$ and $x = 7$

Determine the value of r.

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{7^2 + 3^2}$

$r = \sqrt{58}$

Then P(-7, 3) lies on the terminal arm of α.

$\sin\alpha = \frac{y}{r}$, so $\sin\alpha = \frac{3}{\sqrt{58}}$

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c) The terminal arm lies in Quadrant 4, sin $\theta = \frac{5}{12}$; what is cos α?

Consider point Q from part a.

sin $\theta = \frac{5}{12}$ and sin $\theta = \frac{y}{r}$, so $y = 5$ and $r = 12$

Determine the value of x.

$r^2 = x^2 + y^2$

$12^2 = x^2 + 5^2$

$x = \sqrt{119}$

Then P($\sqrt{119}$, 5) lies on the terminal arm of α.

cos $\alpha = \frac{x}{r}$, so cos $\alpha = \frac{\sqrt{119}}{12}$

14. Which values of θ satisfy each equation for $0° \leq \theta \leq 360°$?

a) tan $\theta = 1$

Use the table on page 444.
$\theta = 45°$ and $\theta = 225°$

b) tan $\theta = 0$

$\theta = 0°$, $\theta = 180°$, and $\theta = 360°$

c) sin $\theta = 1$

Use the table on page 444.
$\theta = 90°$

d) sin $\theta = 0$

$\theta = 0°$, $\theta = 180°$, and $\theta = 360°$

e) cos $\theta = 1$

Use the table on page 444.
$\theta = 0°$ and $\theta = 360°$

f) cos $\theta = 0$

$\theta = 90°$ and $\theta = 270°$

15. Which values of θ satisfy each equation for $0° \leq \theta \leq 360°$?

a) tan $\theta = -1$

Use the table on page 444.
$\theta = 135°$ and $\theta = 315°$

b) sin $\theta = -1$

$\theta = 270°$

c) cos $\theta = -1$

Use the table on page 444.
$\theta = 180°$

16. To the nearest degree, which values of θ satisfy each equation for $0° \leq \theta \leq 360°$?

a) tan $\theta = \frac{1}{2}$

The reference angle is:

$\tan^{-1}\left(\frac{1}{2}\right) \doteq 27°$

tan θ is positive in Quadrant 1, so $\theta \doteq 27°$ or in Quadrant 3, so
$\theta \doteq 180° + 27°$, or $207°$

b) tan $\theta = -\frac{2}{3}$

The reference angle is:

$\tan^{-1}\left(\frac{2}{3}\right) \doteq 34°$

tan θ is negative in Quadrant 2, so
$\theta \doteq 180° - 34°$, or $146°$
or in Quadrant 4, so
$\theta \doteq 360° - 34°$, or $326°$

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c) $\cos \theta = 0.6$

The reference angle is:
$\cos^{-1}(0.6) \doteq 53°$
$\cos \theta$ is positive in Quadrant 1, so $\theta \doteq 53°$ or in Quadrant 4, so
$\theta \doteq 360° - 53°$, or $307°$

d) $\sin \theta = -0.25$

The reference angle is:
$\sin^{-1}(0.25) \doteq 14°$
$\sin \theta$ is negative in Quadrant 3, so
$\theta \doteq 180° + 14°$, or $194°$
or in Quadrant 4, so
$\theta \doteq 360° - 14°$, or $346°$

17. a) Determine sin 40°. For which values of θ, where $0° \leq \theta \leq 360°$, is $\cos \theta = \sin 40°$?

$\sin 40° = 0.6427...$
Solve: $\cos \theta = 0.6427...$
$\theta = \cos^{-1}(0.6427...)$
$\theta = 50°$
$\cos \theta$ is also positive in Quadrant 4.
$\theta = 360° - 50°$, or $310°$

b) Repeat part a for the sines of 4 different angles between 0° and 90°. What patterns do you see in the angles?

Sample response:

$\sin 22° = 0.3746...$
Solve: $\cos \theta = 0.3746...$
$\theta = 68°$
Also, $\theta = 360° - 68°$, or $292°$

$\sin 59° = 0.8571...$
Solve: $\cos \theta = 0.8571...$
$\theta = 31°$
Also, $\theta = 360° - 31°$, or $329°$

$\sin 76° = 0.9702...$
Solve: $\cos \theta = 0.9702...$
$\theta = 14°$
Also, $\theta = 360° - 14°$, or $346°$

$\sin 83° = 0.9925...$
Solve: $\cos \theta = 0.9925...$
$\theta = 7°$
Also, $\theta = 360° - 7°$, or $353°$

When the sine and the cosine of an angle are equal, the angles in Quadrant 1 are complementary.

18. a) Determine tan 40°. For which values of θ, where $0° \leq \theta \leq 360°$, is $\tan \theta = \frac{1}{\tan 40°}$?

$\tan 40° = 0.8390...$
Solve: $\tan \theta = \frac{1}{0.8390...}$, or $1.1917...$
$\theta = \tan^{-1}(1.1917...)$
$\theta = 50°$
$\tan \theta$ is also positive in Quadrant 3.
$\theta = 180° + 50°$, or $230°$

b) Repeat part a for the tangents of 4 different angles between 0° and 90°. What patterns do you see in the angles?

Sample response:

$\tan 22° = 0.4040...$
Solve:
$\tan \theta = \frac{1}{0.4040...}$, or 2.4750...
$\theta = 68°$
Also, $\theta = 180° + 68°$, or 248°

$\tan 59° = 1.6642...$
Solve:
$\tan \theta = \frac{1}{1.6642...}$, or 0.6008...
$\theta = 31°$
Also, $\theta = 180° + 31°$, or 211°

$\tan 76° = 4.0107...$
Solve:
$\tan \theta = \frac{1}{4.0107...}$, or 0.2493...
$\theta = 14°$
Also, $\theta = 180° + 14°$, or 194°

$\tan 83° = 8.1443...$
Solve:
$\tan \theta = \frac{1}{8.1443...}$, or 0.1227...
$\theta = 7°$
Also, $\theta = 180° + 7°$, or 187°

The tangents of complementary angles are reciprocals.

19. For each equation below:

i) To the nearest degree, determine the possible values of θ, for $0° \leq \theta \leq 360°$.

ii) Determine the values of the other two primary trigonometric ratios of each angle θ, to 3 decimal places.

a) $\cos \theta = 0.2$

i) $\cos^{-1}(0.2) \doteq 78°$
$\cos \theta$ is positive in Quadrant 1, so $\theta \doteq 78°$ or in Quadrant 4, so
$\theta \doteq 360° - 78°$, or 282°
ii) $\tan 78° \doteq 4.704$
$\tan 282° \doteq -4.704$
$\sin 78° \doteq 0.978$
$\sin 282° \doteq -0.978$

b) $\sin \theta = 0.9$

i) $\sin^{-1}(0.9) \doteq 64°$
$\sin \theta$ is positive in Quadrant 1, so $\theta \doteq 64°$ or in Quadrant 2, so
$\theta \doteq 180° - 64°$, or 116°
ii) $\tan 64° \doteq 2.050$
$\tan 116° \doteq -2.050$
$\cos 64° \doteq 0.438$
$\cos 116° \doteq -0.438$

c) $\tan \theta = 0.45$

i) $\tan^{-1}(0.45) \doteq 24°$
$\tan \theta$ is positive in Quadrant 1, so $\theta \doteq 24°$ or in Quadrant 3, so
$\theta \doteq 180° + 24°$, or 204°
ii) $\cos 24° \doteq 0.914$
$\cos 204° \doteq -0.914$
$\sin 24° \doteq 0.407$
$\sin 204° \doteq -0.407$

d) $\cos \theta = -0.3$

i) $\cos^{-1}(0.3) \doteq 73°$
$\cos \theta$ is negative in Quadrant 2, so
$\theta \doteq 180° - 73°$, or 107°
or in Quadrant 3, so
$\theta \doteq 180° + 73°$, or 253°
ii) $\sin 107° \doteq 0.956$
$\sin 253° \doteq -0.956$
$\tan 107° \doteq -3.271$
$\tan 253° \doteq 3.271$

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e) $\sin\theta = -0.3$

i) $\sin^{-1}(0.3) \doteq 17°$
sin θ is negative in Quadrant 3, so
$\theta \doteq 180° + 17°$, or 197°
or in Quadrant 4, so
$\theta \doteq 360° - 17°$, or 343°

ii) $\cos 197° \doteq -0.956$
$\cos 343° \doteq 0.956$
$\tan 197° \doteq 0.306$
$\tan 343° \doteq -0.306$

f) $\tan\theta = -1.8$

i) $\tan^{-1}(1.8) \doteq 61°$
tan θ is negative in Quadrant 2, so
$\theta \doteq 180° - 61°$, or 119°
or in Quadrant 4, so
$\theta \doteq 360° - 61°$, or 299°

ii) $\sin 119° \doteq 0.875$
$\sin 299° \doteq -0.875$
$\cos 119° \doteq -0.485$
$\cos 299° \doteq 0.485$

C

20. **a)** For which values of θ is tan θ undefined, where $0° \leq \theta \leq 360°$?

From the table on page 444, tan 90° and tan 270° are undefined.

b) Are there any values of θ, where $0° \leq \theta \leq 360°$, for which sin θ or cos θ are undefined?

Justify your answers.

sin θ and cos θ are defined for all values of θ, where $0° \leq \theta \leq 360°$, because they are ratios with r in the denominator, and $r \neq 0$.

21. To the nearest degree, which values of θ satisfy each equation for $0° \leq \theta \leq 360°$?

a) $\tan\theta = -\tan 60°$

tan θ is negative in Quadrants 2 and 4, so
$\theta = 180° - 60°$, or 120°;
and $\theta = 360° - 60°$, or 300°

b) $\cos\theta = \sin\theta$

In Quadrant 1, where both cos θ and sin θ are positive,
$\theta = 45°$
In Quadrant 3, where both cos θ and sin θ are negative,
$\theta = 225°$

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