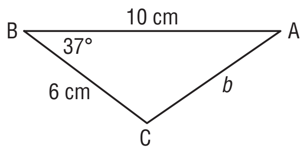
Lesson 6.5 Exercises, pages 498–506

A

3. Which strategy would you use to determine the indicated measure in each triangle?

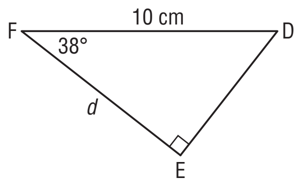
- a primary trigonometric ratio
- the Cosine Law
- the Sine Law

a)



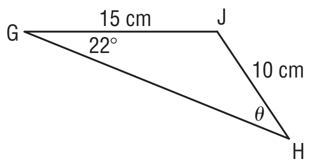
Since 2 sides and the contained angle are given, use the Cosine Law.

b)



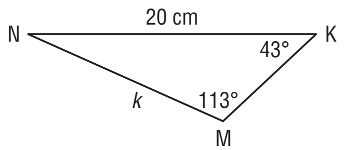
Since it is a right triangle, use a primary trigonometric ratio.

c)



Since 2 sides and a non-contained angle are given, use the Sine Law.

d)



Since 2 angles and a side are given, use the Sine Law.

B

4. Determine each measure in question 3. Give the angles to the nearest degree and the side lengths to the nearest tenth of a unit.

a) Use: $b^2 = a^2 + c^2 - 2ac \cos B$

Substitute: $a = 6$, $c = 10$, $\angle B = 37°$

$b^2 = 6^2 + 10^2 - 2(6)(10) \cos 37°$

$b = \sqrt{6^2 + 10^2 - 2(6)(10) \cos 37°}$

$b = 6.3374...$

$b \doteq 6.3$ cm

b) Use: $\cos 38° = \frac{d}{10}$

$d = 10 \cos 38°$

$d = 7.8801...$

$d \doteq 7.9$ cm

c) Use: $\frac{\sin H}{h} = \frac{\sin G}{g}$

Substitute: $\angle G = 22°$, $h = 15$, $g = 10$

$\frac{\sin H}{15} = \frac{\sin 22°}{10}$

$\sin H = \frac{15 \sin 22°}{10}$

$\angle H = \sin^{-1}\left(\frac{15 \sin 22°}{10}\right)$

$\angle H = 34.1879...°$

$\theta \doteq 34°$

d) Use: $\frac{k}{\sin K} = \frac{m}{\sin M}$

Substitute: $\angle K = 43°$, $\angle M = 113°$, $m = 20$

$\frac{k}{\sin 43°} = \frac{20}{\sin 113°}$

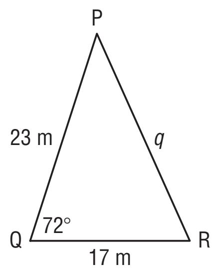
$k = \frac{20 \sin 43°}{\sin 113°}$

$k = 14.8179...$

$k \doteq 14.8$ cm

5. Determine the indicated measure in each triangle. Give the angles to the nearest degree and the side lengths to the nearest tenth of a unit.

a)



Use: $q^2 = p^2 + r^2 - 2pr \cos Q$

Substitute: $p = 17$, $r = 23$, $\angle Q = 72°$

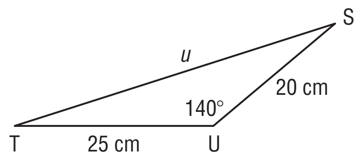
$q^2 = 17^2 + 23^2 - 2(17)(23) \cos 72°$

$q = \sqrt{17^2 + 23^2 - 2(17)(23)\cos 72°}$

$q = 24.0072...$

$q \doteq 24.0$ m

b)



Use: $u^2 = s^2 + t^2 - 2st \cos U$

Substitute: $s = 25$, $t = 20$, $\angle U = 140°$

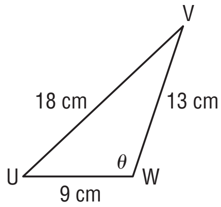
$u^2 = 25^2 + 20^2 - 2(25)(20) \cos 140°$

$u = \sqrt{25^2 + 20^2 - 2(25)(20) \cos 140°}$

$u = 42.3207...$

$u \doteq 42.3$ cm

c)



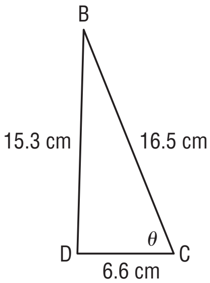
Use: $w^2 = u^2 + v^2 - 2uv \cos W$
Substitute: $w = 18$, $u = 13$, $v = 9$
$18^2 = 13^2 + 9^2 - 2(13)(9) \cos W$

$$\cos W = \frac{13^2 + 9^2 - 18^2}{2(13)(9)}$$

$$\angle W = \cos^{-1}\left(\frac{13^2 + 9^2 - 18^2}{2(13)(9)}\right)$$

$\angle W = 108.4356\ldots°$
$\theta \doteq 108°$

d)



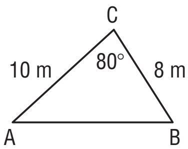
Use: $c^2 = b^2 + d^2 - 2bd \cos C$
Substitute: $c = 15.3$, $b = 6.6$, $d = 16.5$
$15.3^2 = 6.6^2 + 16.5^2 - 2(6.6)(16.5) \cos C$

$$\cos C = \frac{6.6^2 + 16.5^2 - 15.3^2}{2(6.6)(16.5)}$$

$$\angle C = \cos^{-1}\left(\frac{6.6^2 + 16.5^2 - 15.3^2}{2(6.6)(16.5)}\right)$$

$\angle C = 67.9629\ldots°$
$\theta \doteq 68°$

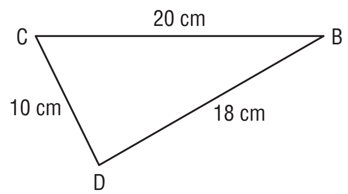
6. A security camera, C, rotates through 80°. The camera is 10 m and 8 m from two doors, A and B, on the same wall of a building. To the nearest metre, how far apart are the doors?



Use: $c^2 = a^2 + b^2 - 2ab \cos C$
Substitute: $a = 8$, $b = 10$, $\angle C = 80°$
$c^2 = 8^2 + 10^2 - 2(8)(10) \cos 80°$
$c = \sqrt{8^2 + 10^2 - 2(8)(10) \cos 80°}$
$c = 11.6711\ldots$
The doors are about 12 m apart.

7. Solve each triangle. Give the side lengths to the nearest tenth of a unit and the angle measures to the nearest degree.

a)



Use: $b^2 = c^2 + d^2 - 2cd \cos B$
Substitute: $b = 10, c = 18, d = 20$
$10^2 = 18^2 + 20^2 - 2(18)(20) \cos B$

$\cos B = \frac{18^2 + 20^2 - 10^2}{2(18)(20)}$

$\angle B = \cos^{-1}\left(\frac{18^2 + 20^2 - 10^2}{2(18)(20)}\right)$

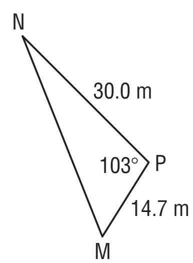
$\angle B = 29.9264\ldots°$
$\angle B \doteq 30°$
Use: $c^2 = b^2 + d^2 - 2bd \cos C$
Substitute: $c = 18, b = 10, d = 20$
$18^2 = 10^2 + 20^2 - 2(10)(20) \cos C$

$\cos C = \frac{10^2 + 20^2 - 18^2}{2(10)(20)}$

$\angle C = \cos^{-1}\left(\frac{10^2 + 20^2 - 18^2}{2(10)(20)}\right)$

$\angle C = 63.8961\ldots°$
$\angle C \doteq 64°$
$\angle D \doteq 180° - (64° + 30°)$
$\angle D \doteq 86°$

b)



Use: $p^2 = m^2 + n^2 - 2mn \cos P$
Substitute: $m = 30, n = 14.7, \angle P = 103°$
$p^2 = 30^2 + 14.7^2 - 2(30)(14.7) \cos 103°$
$p = \sqrt{30^2 + 14.7^2 - 2(30)(14.7) \cos 103°}$
$p = 36.2559\ldots$

Use: $\frac{\sin M}{m} = \frac{\sin P}{p}$

Substitute: $\angle P = 103°, m = 30, p = 36.2559\ldots$

$\frac{\sin M}{30} = \frac{\sin 103°}{36.2559\ldots}$

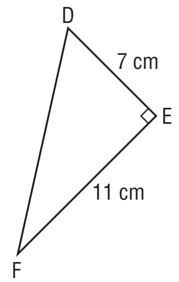
$\sin M = \frac{30 \sin 103°}{36.2559\ldots}$

Since $\angle M$ is acute:

$\angle M = \sin^{-1}\left(\frac{30 \sin 103°}{36.2559\ldots}\right)$

$\angle M = 53.7303\ldots°$
$\angle N = 180° - (103° + 53.7303\ldots°)$
$\angle N = 23.2696\ldots°$
So, $MN \doteq 36.3$ m, $\angle M \doteq 54°$, $\angle N \doteq 23°$

8. a) Use the Cosine Law to determine the length of DF to the nearest tenth of a centimetre.



Use: $e^2 = d^2 + f^2 - 2df \cos E$
Substitute: $d = 11$, $f = 7$, $\angle E = 90°$
$e^2 = 11^2 + 7^2 - 2(11)(7) \cos 90°$
$e = \sqrt{11^2 + 7^2}$
$e = 13.0384\ldots$
DF $\doteq$ 13.0 cm

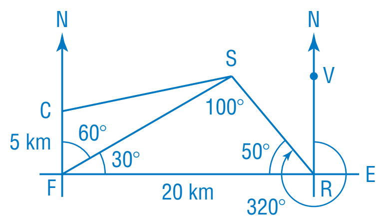
b) How is the Pythagorean Theorem a special case of the Cosine Law?

When the angle in the statement of the Cosine Law is 90°, since $\cos 90° = 0$, the Cosine Law reduces to the Pythagorean Theorem.

9. A fire spotter sees smoke on a bearing of 060°. At a point 20 km due east of the fire spotter, a ranger sees the same smoke on a bearing of 320°.

a) How far is the smoke from each location?

Sketch a diagram.
$\angle SRV$ is: $360° - 320° = 40°$
In ΔSFR,
$\angle F$ is: $90° - 60° = 30°$
$\angle R$ is: $90° - 40° = 50°$
$\angle S$ is: $180° - (30° + 50°) = 100°$



Use: $\dfrac{f}{\sin F} = \dfrac{s}{\sin S}$
Substitute: $\angle F = 30°$, $\angle S = 100°$, $s = 20$
$\dfrac{f}{\sin 30°} = \dfrac{20}{\sin 100°}$
$f = \dfrac{20 \sin 30°}{\sin 100°}$
$f = 10.1542\ldots$

Use: $\dfrac{r}{\sin R} = \dfrac{20}{\sin 100°}$
Substitute: $\angle R = 50°$
$\dfrac{r}{\sin 50°} = \dfrac{20}{\sin 100°}$
$r = \dfrac{20 \sin 50°}{\sin 100°}$
$r = 15.5572\ldots$

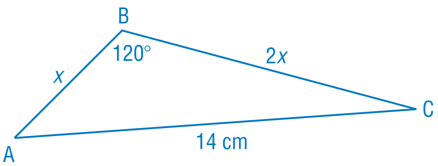
The smoke is approximately 10 km and 16 km from each location.

b) A fire crew is 5 km due north of the fire spotter. How far is the crew from the smoke?
Give the answers to the nearest kilometre.

In ΔSFC,
Use: $f^2 = c^2 + s^2 - 2cs \cos F$
Substitute: $c = 15.5572\ldots$, $s = 5$, $\angle F = 60°$
$f^2 = 15.5572\ldots^2 + 5^2 - 2(15.5572\ldots)(5) \cos 60°$
$f = \sqrt{15.5572\ldots^2 + 5^2 - 2(15.5572\ldots)(5) \cos 60°}$
$f = 13.7565\ldots$
The crew is approximately 14 km from the fire.

10. In ΔABC, BC = 2AB, ∠B = 120°, and AC = 14 cm; determine the exact lengths of AB and BC.

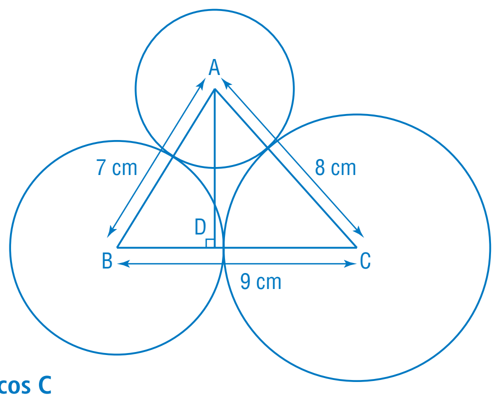
Sketch a diagram.
Let AB = x centimetres
Then BC = $2x$ centimetres
Use: $b^2 = a^2 + c^2 - 2ac \cos B$
Substitute:



$b = 14$, $a = 2x$, $c = x$, $\angle B = 120°$
$14^2 = (2x)^2 + x^2 - 2(2x)(x) \cos 120°$
$196 = 5x^2 - 4x^2(-0.5)$
$196 = 7x^2$
$x^2 = 28$
$x = \sqrt{28}$, or $2\sqrt{7}$
So, AB = $2\sqrt{7}$ cm and AC = $4\sqrt{7}$ cm

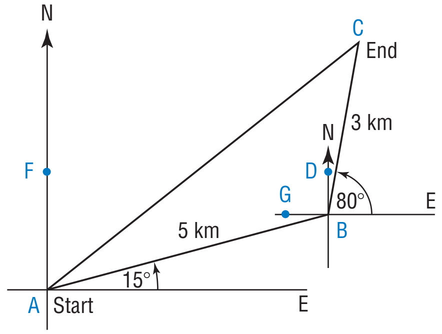
11. Three circles have radii 3 cm, 4 cm, and 5 cm. Each circle just touches the other 2 circles externally. To the nearest tenth of a square centimetre, determine the area of the triangle formed by the centres of the circles.

Sketch a diagram.
The centres of the circle form ΔABC, with:
AB: 3 cm + 4 cm = 7 cm
BC: 4 cm + 5 cm = 9 cm
AC: 3 cm + 5 cm = 8 cm
Draw the perpendicular from A to meet BC at D.



In ΔABC, use:
$c^2 = a^2 + b^2 - 2ab \cos C$
Substitute: $c = 7$, $a = 9$, $b = 8$
$7^2 = 9^2 + 8^2 - 2(9)(8) \cos C$
$144 \cos C = 96$
$\cos C = \frac{96}{144}$
$\angle C = 48.1896...°$
In ΔACD, use:
$\sin C = \frac{AD}{AC}$
$AD = 8 \sin 48.1896...°$
$AD = 5.9628...$
The area of ΔABC is:
$0.5(BC)(AD) = 0.5(9)(5.9628...)$
$= 26.8328...$
So, the area of the triangle formed by the centres of the circles is approximately 26.8 cm^2.

12. Here is a sketch of the route taken by an orienteering group on a one-day trek.



a) To the nearest tenth of a kilometre, what is the straight-line distance from the start to the end?

$\angle$DBC is: $90° - 80° = 10°$
$\angle$GBA $= 15°$
So, $\angle$ABC is: $15° + 90° + 10° = 115°$
In ΔABC, use: $b^2 = a^2 + c^2 - 2ac \cos B$
Substitute: $a = 3$, $c = 5$, $\angle B = 115°$
$b^2 = 3^2 + 5^2 - 2(3)(5) \cos 115°$
$b = \sqrt{3^2 + 5^2 - 2(3)(5) \cos 115°}$
$b = 6.8321...$
The straight-line distance is approximately 6.8 km.

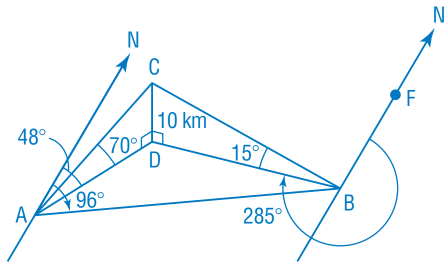
b) To the nearest degree, what is the bearing of the end point from the start point?

In ΔABC, determine $\angle$A; use:
$\frac{\sin A}{a} = \frac{\sin B}{b}$ Substitute: $\angle B = 115°$, $a = 3$, $b = 6.8321...$
$\frac{\sin A}{3} = \frac{\sin 115°}{6.8321...}$
$\sin A = \frac{3 \sin 115°}{6.8321...}$
Since $\angle$A is acute:
$\angle A = \sin^{-1}\left(\frac{3 \sin 115°}{6.8321...}\right)$
$\angle A = 23.4506...°$
$\angle A \doteq 23°$
So, $\angle$FAC is approximately $90° - (23° + 15°) = 52°$
The bearing of the end point from the start point is approximately 052°.

C

13. Observers on two ships see an aircraft flying at 10 000 m. Observer A reports the bearing of the aircraft as 048° and its angle of elevation as 70°. Observer B reports the bearing of the aircraft as 285° and its angle of elevation as 15°. The bearing of observer B from observer A is 096°.
To the nearest tenth of a kilometre, how far apart are the ships?

Sketch a diagram.



The diagram is not drawn to scale.

In ΔACD,

$\angle C = 90° - 70°$
$= 20°$

Determine AD; use:

$\tan 20° = \frac{AD}{10}$

$AD = 10 \tan 20°$

In ΔBCD,

$\angle C = 90° - 15°$
$= 75°$

Determine BD; use:

$\tan 75° = \frac{BD}{10}$

$BD = 10 \tan 75°$

$\angle FBA = 180° - 96°$
$= 84°$

$\angle FBD = 360° - 285°$
$= 75°$

So, $\angle ABD = 84° - 75°$
$= 9°$

In ΔABD,

$\angle A = 96° - 48°$
$= 48°$

$\angle B = 9°$

So, $\angle D = 180° - (48° + 9°)$
$= 123°$

Use: $d^2 = a^2 + b^2 - 2ab \cos D$

Substitute: $a = 10 \tan 75°$, $b = 10 \tan 20°$, $\angle D = 123°$

$d^2 = (10 \tan 75°)^2 + (10 \tan 20°)^2 - 2(10 \tan 75°)(10 \tan 20°) \cos 123°$

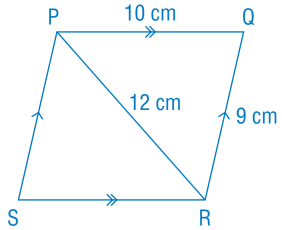
$d^2 = 10^2[(\tan 75°)^2 + (\tan 20°)^2 - 2(\tan 75°)(\tan 20°) \cos 123°]$

$d = 10\sqrt{(\tan 75°)^2 + (\tan 20°)^2 - 2(\tan 75°)(\tan 20°) \cos 123°}$

$d = 39.4211...$

The ships are approximately 39.4 km apart.

14. The lengths of the sides of a parallelogram are 9 cm and 10 cm. The shorter diagonal is 12 cm long. To the nearest tenth of a centimetre, what is the length of the longer diagonal?



Sketch a diagram.

In ΔPQR, determine $\angle Q$.

Use: $q^2 = p^2 + r^2 - 2pr \cos Q$

Substitute: $q = 12$, $p = 9$, $r = 10$

$12^2 = 9^2 + 10^2 - 2(9)(10) \cos Q$

$180 \cos Q = 37$

$\cos Q = \frac{37}{180}$

$\angle Q = 78.1379...°$

So, $\angle QPS = 180° - 78.1379...°$

$= 101.8620...°$

In ΔPQS, determine QS.

Use: $p^2 = q^2 + s^2 - 2qs \cos P$

Substitute: $q = 9$, $s = 10$, $\angle P = 101.8620...°$

$p^2 = 9^2 + 10^2 - 2(9)(10) \cos 101.8620...°$

$p = \sqrt{9^2 + 10^2 - 2(9)(10) \cos 101.8620...°}$

$p = 14.7648...$

The longer diagonal is approximately 14.8 cm long.