PRACTICE TEST, pages 516–520

1. Multiple Choice The terminal arm of an angle θ in standard position is in Quadrant 4. Which statements are true?

I. $\tan\theta < 0$ **II.** $\tan\theta > 0$ **III.** $\sin\theta < \cos\theta$ **IV.** $\sin\theta > \cos\theta$

A. I and III **B.** I and IV **C.** II and III **D.** II and IV

2. Multiple Choice An angle θ is in standard position, with $\sin\theta = \frac{1}{5}$. Which statement could be true?

A. $\cos\theta = \frac{4}{5}$ **B.** $\cos\theta = \frac{2\sqrt{6}}{5}$ **C.** $\cos\theta = \frac{\sqrt{26}}{5}$ **D.** $\cos\theta = \frac{2}{5}$

3. Point P(−2, 7) is on the terminal arm of an angle θ in standard position.

a) Determine $\sin\theta$, $\cos\theta$, and $\tan\theta$.

The distance of P from the origin is r.
Use: $r = \sqrt{x^2 + y^2}$ Substitute: $x = -2$, $y = 7$

$r = \sqrt{(-2)^2 + 7^2}$

$r = \sqrt{53}$

$\sin\theta = \frac{y}{r} = \frac{7}{\sqrt{53}}$

$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{53}}$

$\tan\theta = \frac{y}{x} = \frac{7}{-2}$, or -3.5

b) Determine the measure of θ to the nearest degree.

Use: $\sin\theta = \frac{7}{\sqrt{53}}$
The reference angle is:

$\sin^{-1}\left(\frac{7}{\sqrt{53}}\right) \doteq 74°$

θ is approximately $180° - 74°$, or $106°$.

4. Without using a calculator, determine the exact value of each expression.

a) $(\sin 45°)(\cos 45°)$

Substitute: $\sin 45° = \frac{1}{\sqrt{2}}$,

$\cos 45° = \frac{1}{\sqrt{2}}$

$(\sin 45°)(\cos 45°)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{2}$

b) $\frac{\tan 45°}{\cos 60°}$

Substitute: $\tan 45° = 1$,
$\cos 60° = 0.5$

$\frac{\tan 45°}{\cos 60°} = \frac{1}{0.5}$

$= 2$

c) (tan 120°)(tan 210°)

Substitute: $\tan 120° = -\sqrt{3}$,

$\tan 210° = \frac{1}{\sqrt{3}}$

(tan 120°)(tan 210°)

$= (-\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)$

$= -1$

d) sin 330° + cos 120°

Substitute: sin 330° = −0.5,

cos 120° = −0.5

sin 330° + cos 120°

$= -0.5 + (-0.5)$

$= -1$

5. For which angles in standard position are the following statements true? Give the angle measures to the nearest degree for $0° \leq \theta \leq 360°$.

a) $\sin \theta = -\frac{1}{\sqrt{2}}$

The reference angle is:

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

$\sin \theta$ is negative in Quadrants 3 and 4, so

$\theta = 180° + 45°$, or 225°

or

$\theta = 360° - 45°$, or 315°

b) $\cos \theta = \frac{3}{4}$

The reference angle is:

$\cos^{-1}\left(\frac{3}{4}\right) \doteq 41°$

$\cos \theta$ is positive in Quadrants 1 and 4, so

$\theta \doteq 41°$

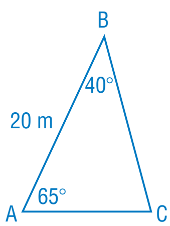
or

$\theta \doteq 360° - 41°$, or 319°

6. Solve each triangle. Give the angle measures to the nearest degree and the side lengths to the nearest tenth of a unit.

a) In ΔABC, AB = 20 m, ∠A = 65°, and ∠B = 40°

Sketch a diagram. Since 2 angles and the contained side are given, only one triangle can be drawn.



$\angle C = 180° - (40° + 65°)$

$= 75°$

Use: $\frac{a}{\sin A} = \frac{c}{\sin C}$

Substitute: ∠A = 65°

∠C = 75°, $c = 20$

$\frac{a}{\sin 65°} = \frac{20}{\sin 75°}$

$a = \frac{20 \sin 65°}{\sin 75°}$

$a = 18.7655\ldots$

Use: $\frac{b}{\sin B} = \frac{20}{\sin 75°}$

Substitute: ∠B = 40°

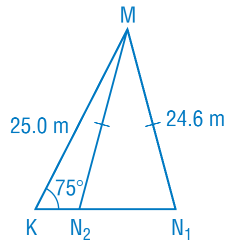
$b = \frac{20 \sin 40°}{\sin 75°}$

$b = 13.3092\ldots$

So, BC ≐ 18.8 m and AC ≐ 13.3 m

b) In ΔKMN, KM = 25.0 m, MN = 24.6 m, and $\angle$K = 75°

Check for the ambiguous case.
The ratio of the side opposite $\angle$K to the side adjacent to $\angle$K is:
$\frac{MN}{KM} = \frac{24.6}{25.0}$, which is 0.984
$\sin 75° = 0.9659...$
Since $\sin 75° < 0.984 < 1$, then there is an ambiguous case, and 2 triangles can be constructed: ΔKMN_1 is acute; ΔKMN_2 is obtuse.
Sketch a diagram.



This diagram is not drawn to scale.

In ΔKMN_1

Determine $\angle N_1$.

Use: $\frac{\sin N_1}{n} = \frac{\sin K}{k}$

Substitute: $\angle K = 75°$, $n = 25$, $k = 24.6$

$\frac{\sin N_1}{25} = \frac{\sin 75°}{24.6}$

$\sin N_1 = \frac{25 \sin 75°}{24.6}$

$\angle N_1 = \sin^{-1}\left(\frac{25 \sin 75°}{24.6}\right)$

$\angle N_1 = 79.0014...°$

So, $\angle M = 180° - (75° + 79.0014...°)$

$= 25.9985...°$

Use: $\frac{m}{\sin M} = \frac{k}{\sin K}$

Substitute: $\angle M = 25.9985...°$

$\angle K = 75°$, $k = 24.6$

$\frac{m}{\sin 25.9985...°} = \frac{24.6}{\sin 75°}$

$m = \frac{24.6 \sin 25.9985...°}{\sin 75°}$

$m = 11.1637...$

So, $\angle N \doteq 79°$, $\angle M \doteq 26°$, and KN $\doteq$ 11.2 m

In ΔKMN_2

$\angle N_2 = 180° - \angle N_1$

$= 100.9985...°$

So, $\angle M = 180° - (75° + 100.9985...°)$

$= 4.0014...°$

Use: $\frac{m}{\sin M} = \frac{k}{\sin K}$

Substitute: $\angle M = 4.0014...°$,

$\angle K = 75°$, $k = 24.6$

$\frac{m}{\sin 4.0014...°} = \frac{24.6}{\sin 75°}$

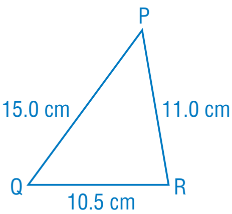
$m = \frac{24.6 \sin 4.0014...°}{\sin 75°}$

$m = 1.7771...$

So, $\angle N \doteq 101°$, $\angle M \doteq 4°$, and KN $\doteq$ 1.8 m

c) In ΔPQR, PQ = 15.0 cm, PR = 11.0 cm, and QR = 10.5 cm

Sketch a diagram. Since 3 sides are given, only one triangle can be drawn.



Use: $r^2 = p^2 + q^2 - 2pq \cos R$

Substitute: $r = 15$, $p = 10.5$, $q = 11$

$15^2 = 10.5^2 + 11^2 - 2(10.5)(11) \cos R$

$231 \cos R = 6.25$

$\cos R = \frac{6.25}{231}$

$\angle R = 88.4496...°$

Use: $q^2 = p^2 + r^2 - 2pr \cos Q$

Substitute the values above.

$11^2 = 10.5^2 + 15^2 - 2(10.5)(15) \cos Q$

$315 \cos Q = 214.25$

$\cos Q = \frac{214.25}{315}$

$\angle Q = 47.1439...°$

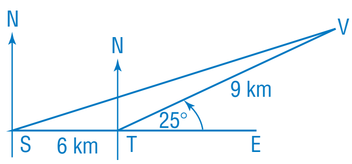
$\angle P = 180° - (88.4496...° + 47.1439...°)$

$= 44.4064...°$

So, $\angle P \doteq 44°$, $\angle Q \doteq 47°$, $\angle R \doteq 88°$

7. A cross-country runner runs due east for 6 km, then changes course to E25°N and runs another 9 km. To the nearest tenth of a kilometre, how far is the runner from her starting point?

Sketch a diagram.



In ΔSTV,

$\angle T = 180° - 25°$

$= 155°$

To determine SV, use:

$t^2 = s^2 + v^2 - 2sv \cos T$

Substitute: $s = 9$, $v = 6$, $\angle T = 155°$

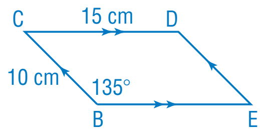
$t^2 = 9^2 + 6^2 - 2(9)(6) \cos 155°$

$t = \sqrt{9^2 + 6^2 - 2(9)(6) \cos 155°}$

$t = 14.6588...$

The runner is approximately 14.7 km from her starting point.

8. In parallelogram BCDE, BC = 10 cm, CD = 15 cm, and ∠B = 135°; determine the lengths of the diagonals to the nearest tenth of a centimetre.



Sketch a diagram.

$\angle C = 180° - 135°$

$= 45°$

In ΔBCD, to determine BD, use:

$c^2 = b^2 + d^2 - 2bd \cos C$

Substitute: $b = 15$, $d = 10$, $\angle C = 45°$

$c^2 = 15^2 + 10^2 - 2(15)(10) \cos 45°$

$c = \sqrt{15^2 + 10^2 - 2(15)(10) \cos 45°}$

$c = 10.6239...$

Diagonal BD is approximately 10.6 cm long.

In ΔBCE, to determine CE, use:

$b^2 = c^2 + e^2 - 2ce \cos B$

Substitute: $c = 15$, $e = 10$, $\angle B = 135°$

$b^2 = 15^2 + 10^2 - 2(15)(10) \cos 135°$

$b = \sqrt{15^2 + 10^2 - 2(15)(10) \cos 135°}$

$b = 23.1761...$

Diagonal CE is approximately 23.2 cm long.