Lesson 7.1 Exercises, pages 527–531

A

4. Which expressions are rational expressions? Justify your choices.

a) $\frac{x + 3}{5}$ b) $\frac{\sqrt{x} + 2}{4x}$ c) $\frac{a}{7}$

d) $\frac{3^x + 9}{x^2 - 2}$ e) $\frac{3\sqrt[3]{b} + 2b}{16 - b^2}$ f) $\frac{x^2 + 2x - 7}{x + 3}$

Parts a, c, and f are rational expressions because each expression is the quotient of two polynomials. Parts b and e are not rational expressions because they each contain the root of a variable; part d is not a rational expression because it has a variable as an exponent.

5. Identify the non-permissible values of the variable for each rational expression.

a) $\frac{2 - x^2}{x + 5}$

$x + 5 = 0$
$x = -5$
So, $x = -5$ is the non-permissible value.

b) $\frac{x + 1}{(x - 2)(x + 8)}$

$(x - 2)(x + 8) = 0$
$x - 2 = 0$ or $x + 8 = 0$
$x = 2$ or $x = -8$
So, $x = 2$ and $x = -8$ are the non-permissible values.

6. Determine whether the given value of x is a non-permissible value for the rational expression. Explain how you know.

a) $\frac{3x + 9}{(x - 5)(x + 6)}$; $x = 6$

No, $x = 6$ does not result in the denominator equal to 0.

b) $\frac{5x}{x^2 - 4}$; $x = -2$

Yes, $x = -2$ results in the denominator equal to 0.

7. Simplify each rational expression. State the non-permissible values of the variables.

a) $\frac{25mn}{5m}$

The non-permissible value is: $m = 0$

$\frac{25mn}{5m} = \frac{^{5}\cancel{25}\,\cancel{m}\,n}{\cancel{5}\,\cancel{m}}$

$= 5n, m \neq 0$

b) $\frac{2x(x - 3)}{x - 3}$

The non-permissible value is: $x = 3$

$\frac{2x(x - 3)}{x - 3} = \frac{2x\cancel{(x - 3)}}{\cancel{x - 3}}$

$= 2x, x \neq 3$

c) $\frac{(x - 3)(x + 4)}{(x + 4)(x + 6)}$

The non-permissible values are: $x = -4$ and $x = -6$

$\frac{(x - 3)(x + 4)}{(x + 4)(x + 6)}$

$= \frac{(x - 3)\cancel{(x + 4)}}{\cancel{(x + 4)}(x + 6)}$

$= \frac{x - 3}{x + 6}, x \neq -6, -4$

d) $\frac{3x}{12x(x + 5)}$

The non-permissible values are: $x = 0$ and $x = -5$

$\frac{3x}{12x(x + 5)}$

$= \frac{\cancel{3x}}{_{4}\cancel{12x}(x + 5)}$

$= \frac{1}{4(x + 5)}, x \neq -5, 0$

B

8. Determine the non-permissible values for each rational expression.

a) $\frac{x^2 + 3}{x^2 - x - 20}$

$x^2 - x - 20 = 0$

$(x - 5)(x + 4) = 0$

So, $x = 5$ and $x = -4$ are the non-permissible values.

b) $\frac{3x}{x^2 - 36}$

$x^2 - 36 = 0$

$(x - 6)(x + 6) = 0$

So, $x = -6$ and $x = 6$ are the non-permissible values.

c) $\frac{x(2x - 3)}{4x^2 + 17x - 15}$

$4x^2 + 17x - 15 = 0$

$4x^2 + 20x - 3x - 15 = 0$

$4x(x + 5) - 3(x + 5) = 0$

$(4x - 3)(x + 5) = 0$

So, $x = -5$ and $x = \frac{3}{4}$ are the non-permissible values.

d) $\frac{2x}{12x^2 + 2x}$

$12x^2 + 2x = 0$

$2x(6x + 1) = 0$

So, $x = -\frac{1}{6}$ and $x = 0$ are the non-permissible values.

9. Which of these rational expressions are defined for all real values of x? Explain how you know.

a) $\frac{2x^3 + 3}{6x}$

$x = 0$ is a non-permissible value, so the expression is not defined for all values of x.

b) $\frac{3x + 7}{x^2 - 9}$

$x = 3$ and $x = -3$ are non-permissible values, so the expression is not defined for all values of x.

c) $\frac{3x^2 + 2x - 1}{x^2 + 49}$

Since $x^2 \geq 0$, $x^2 + 49 > 0$
Since the denominator cannot equal 0, the expression is defined for all values of x.

d) $\frac{x^2 + 4}{x^3 + 1}$

$x = -1$ is a non-permissible value, so the expression is not defined for all values of x.

10. Use multiplication and division to write two equivalent forms of each rational expression.

a) $\frac{2x}{2x + 4}$

The expression has $x = -2$ as a non-permissible value.

$$\frac{2x}{2x + 4} = \frac{2x}{(2x + 4)} \cdot \frac{(x + 1)}{(x + 1)}$$
$$= \frac{2x(x + 1)}{(2x + 4)(x + 1)}$$

This expression has an additional non-permissible value: $x = -1$

$$\frac{2x}{2x + 4} = \frac{\cancel{2}x}{\cancel{2}(x + 2)}$$
$$= \frac{x}{x + 2}$$

The equivalent expressions are:

$\frac{2x(x + 1)}{(2x + 4)(x + 1)}$, $x \neq -2, -1$

$\frac{x}{x + 2}$, $x \neq -2$

b) $\frac{3(x + 5)}{x(x + 8)(x + 5)}$

This expression has $x = 0$, $x = -8$, and $x = -5$ as non-permissible values.

$$\frac{3(x + 5)}{x(x + 8)(x + 5)}$$
$$= \frac{3(x + 5)}{x(x + 8)(x + 5)} \cdot \frac{(x + 5)}{(x + 5)}$$
$$= \frac{3(x + 5)^2}{x(x + 8)(x + 5)^2}$$

$$\frac{3(x + 5)}{x(x + 8)(x + 5)}$$
$$= \frac{3\cancel{(x + 5)}}{x(x + 8)\cancel{(x + 5)}}$$
$$= \frac{3}{x(x + 8)}$$

The equivalent expressions are:

$\frac{3(x + 5)^2}{x(x + 8)(x + 5)^2}$, $x \neq -8, -5, 0$

and $\frac{3}{x(x + 8)}$, $x \neq -8, -5, 0$

11. **a)** Write each rational expression in simplest form.

i) $\frac{-p^3q^2}{5p^2q^2}$

The non-permissible values are: $p = 0$ and $q = 0$

$\frac{-p^3q^2}{5p^2q^2} = \frac{-p^{\cancel{3}1}\cancel{q^2}}{5\cancel{p^2}\cancel{q^2}}$

$= \frac{-p}{5}, p \neq 0, q \neq 0$

ii) $\frac{4x - 9}{x^2 - 9}$

The non-permissible values are: $x = -3$ and $x = 3$

The numerator and denominator have no common factors, so the expression is in simplest form:

$\frac{4x - 9}{x^2 - 9}, x \neq -3, 3$

iii) $\frac{2x^3 + 4x^2}{6x^2 - 24}$

$= \frac{2x^2(x + 2)}{6(x^2 - 4)}$

$= \frac{2x^2(x + 2)}{6(x - 2)(x + 2)}$

The non-permissible values are:
$x = -2$ and $x = 2$

$= \frac{\cancel{2}x^2\cancel{(x + 2)}}{_3\cancel{6}(x - 2)\cancel{(x + 2)}}$

$= \frac{x^2}{3(x - 2)}, x \neq -2, 2$

iv) $\frac{36 - 9x^2}{x^2 - 5x + 6}$

$= \frac{9(4 - x^2)}{(x - 2)(x - 3)}$

$= \frac{9(2 - x)(2 + x)}{(x - 2)(x - 3)}$

The non-permissible values are:
$x = 2$ and $x = 3$

$= \frac{-9\cancel{(x - 2)}(2 + x)}{\cancel{(x - 2)}(x - 3)}$

$= \frac{-9(2 + x)}{x - 3}, x \neq 2, 3$

b) Choose one expression from part a. Explain why the non-permissible values of the given expression and its simplest form are the same.

In part iii, the numerator and denominator of the expression were divided by the common factor $x + 2$. This division is not possible when $x = -2$. So, $x = -2$ must be included in the non-permissible values of the simplified expression.

12. Here is a student's solution for simplifying a rational expression. Identify the error in the solution. Write a correct solution.

$$\frac{3x - 12}{x^2 + x - 20} = \frac{3\cancel{(x - 4)}}{(x + 5)\cancel{(x - 4)}}$$
$$= \frac{3}{x + 5}, x \neq -5$$

The error in the solution is that $x = 4$ should be included as a non-permissible value. Division by $x - 4$ is not possible when $x = 4$.

Correct solution:

$$\frac{3x - 12}{x^2 + x - 20} = \frac{3\cancel{(x - 4)}}{(x + 5)\cancel{(x - 4)}}$$
$$= \frac{3}{(x + 5)}, x \neq -5, 4$$

13. A student claims that the expressions $\frac{12x^2}{15x}$ and $\frac{12x^2(x - 3)}{15x(x - 3)}$ are equivalent. Is the student correct? Explain.

No, the student is not correct. When $x = 3$, the first expression equals 2.4 and the second expression is not defined. So, the expressions are not equivalent. The non-permissible values of both expressions must be stated.

14. Create a rational expression that has each set of non-permissible values. Explain how you created each expression.

a) $x \neq 0, x \neq 6$

Choose a denominator so that when $x = 0$ or 6, the value of the denominator is 0, and the value of the denominator is non-zero for all other values of x; for example, $x(x - 6)$

Then write a polynomial in the numerator: $\frac{x^2 + 1}{x(x - 6)}$

b) $x \neq 4, x \neq -7$

Choose a denominator so that when $x = 4$ or -7, the value of the denominator is 0, and the value of the denominator is non-zero for all other values of x; for example, $(x - 4)(x + 7)$

Then write a polynomial in the numerator: $\frac{-2x^2 + x}{(x - 4)(x + 7)}$

C

15. Write each rational expression in simplest form. State the non-permissible values of the variables.

a) $\frac{2x^2 - 7xy + 6y^2}{x^4 - 16y^4}$

$$= \frac{2x^2 - 4xy - 3xy + 6y^2}{(x^2 - 4y^2)(x^2 + 4y^2)}$$

$$= \frac{2x(x - 2y) - 3y(x - 2y)}{(x - 2y)(x + 2y)(x^2 + 4y^2)}$$

$$= \frac{(2x - 3y)(x - 2y)}{(x - 2y)(x + 2y)(x^2 + 4y^2)}$$

The non-permissible values are: $x = 2y$ and $x = -2y$

$$= \frac{(2x - 3y)\cancel{(x - 2y)}}{\cancel{(x - 2y)}(x + 2y)(x^2 + 4y^2)}$$

$$= \frac{2x - 3y}{(x + 2y)(x^2 + 4y^2)}, x \neq 2y, -2y$$

b) $\frac{x^4 - 5x^2 + 4}{x^3 + 3x^2 + 2x}$

$$= \frac{(x^2 - 4)(x^2 - 1)}{x(x^2 + 3x + 2)}$$

$$= \frac{(x - 2)(x + 2)(x - 1)(x + 1)}{x(x + 1)(x + 2)}$$

The non-permissible values are: $x = 0$, $x = -2$, and $x = -1$

$$= \frac{(x - 2)\cancel{(x + 2)}(x - 1)\cancel{(x + 1)}}{x\cancel{(x + 1)}\cancel{(x + 2)}}$$

$$= \frac{(x - 2)(x - 1)}{x}, x \neq -2, -1, 0$$