Lesson 7.2 Exercises, pages 537–543

A

3. Simplify each expression.

a) $\frac{3}{4} \cdot \frac{2b}{5}$

$= \frac{3}{\cancel{4}_{2}} \cdot \frac{\cancel{2}b}{5}$

$= \frac{3b}{10}$

b) $\frac{p^2}{5} \cdot \frac{15}{p}$

Non-permissible value: $p = 0$

$\frac{p^2}{5} \cdot \frac{15}{p} = \frac{p^{\cancel{2}}}{\cancel{5}} \cdot \frac{\cancel{15}^{3}}{\cancel{p}}$

$= 3p, p \neq 0$

c) $\frac{-4x}{3} \cdot \frac{9x}{2}$

$= \frac{-^{2}\cancel{4}x}{\cancel{3}} \cdot \frac{^{3}\cancel{9}x}{\cancel{2}}$

$= -6x^2$

d) $\frac{c^3}{10} \cdot \frac{-5}{2c}$

Non-permissible value: $c = 0$

$\frac{c^3}{10} \cdot \frac{-5}{2c} = \frac{c^{\cancel{3}^{2}}}{\cancel{10}_{2}} \cdot \frac{-\cancel{5}}{2\cancel{c}}$

$= \frac{-c^2}{4}, c \neq 0$

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4. Simplify each expression.

a) $\frac{2}{n} \div \frac{4}{n}$

Non-permissible value: $n = 0$

$\frac{2}{n} \div \frac{4}{n} = \frac{2}{n} \cdot \frac{n}{4}$

$= \frac{1}{2}, n \neq 0$

b) $\frac{2b}{5} \div \frac{4}{10b}$

Non-permissible value: $b = 0$

$\frac{2b}{5} \div \frac{4}{10b} = \frac{2b}{5} \cdot \frac{10b}{4}$

$= b^2, b \neq 0$

c) $\frac{6}{x} \div 3$

Non-permissible value: $x = 0$

$\frac{6}{x} \div 3 = \frac{6}{x} \cdot \frac{1}{3}$

$= \frac{2}{x}, x \neq 0$

d) $\frac{d^3}{4} \div \frac{d}{12}$

Non-permissible value: $d = 0$

$\frac{d^3}{4} \div \frac{d}{12} = \frac{d^3}{4} \cdot \frac{12}{d}$

$= 3d^2, d \neq 0$

5. Simplify each expression.

a) $\frac{m}{5p} \cdot \frac{10p^2}{3m}$

Non-permissible values:
$p = 0$ and $m = 0$

$\frac{m}{5p} \cdot \frac{10p^2}{3m} = \frac{m}{5p} \cdot \frac{10p^2}{3m}$

$= \frac{2p}{3}, p \neq 0, m \neq 0$

b) $\frac{-2ab^2}{5a^3b} \div \frac{-4a}{15ab}$

Non-permissible values:
$a = 0$ and $b = 0$

$\frac{-2ab^2}{5a^3b} \div \frac{-4a}{15ab} = \frac{-2ab^2}{5a^3b} \cdot \frac{15ab}{-4a}$

$= \frac{3b^2}{2a^2}, a \neq 0, b \neq 0$

B

6. Simplify each expression.

a) $\frac{3x}{2(x-3)} \cdot \frac{8(x-3)}{9x^2}$

Non-permissible values:
$x = 0$ and $x = 3$

$= \frac{3x}{2(x-3)} \cdot \frac{8(x-3)}{9x^2}$

$= \frac{4}{3x}, x \neq 0, 3$

b) $\frac{15(x+5)}{2x^2} \cdot \frac{8x}{5(x+5)^2}$

Non-permissible values:
$x = 0$ and $x = -5$

$= \frac{15(x+5)}{2x^2} \cdot \frac{8x}{5(x+5)^2}$

$= \frac{12}{x(x+5)}, x \neq -5, 0$

c) $\frac{2(x-4)(x+5)}{3(x+1)} \div \frac{4(x+5)}{(x+1)^2}$

Non-permissible values:

$x = -5$ and $x = -1$

$= \frac{2(x-4)(x+5)}{3(x+1)} \cdot \frac{(x+1)^2}{4(x+5)}$

$= \frac{(x-4)(x+1)}{6}, x \neq -5, -1$

d) $\frac{10y}{(y-3)^3} \div \frac{y(y+1)}{(y-3)^2}$

Non-permissible values:

$y = 3$, $y = 0$, and $y = -1$

$= \frac{10y}{(y-3)^3} \cdot \frac{(y-3)^2}{y(y+1)}$

$= \frac{10}{(y-3)(y+1)}, y \neq -1, 0, 3$

7. Simplify each expression.

a) $\frac{2a+2b}{3a-6b} \cdot \frac{a-2b}{a+b}$

Non-permissible values:

$a = 2b$ and $a = -b$

$\frac{2a+2b}{3a-6b} \cdot \frac{a-2b}{a+b}$

$= \frac{2(a+b)}{3(a-2b)} \cdot \frac{a-2b}{a+b}$

$= \frac{2}{3}, a \neq 2b, -b$

b) $\frac{n^2-4}{2n+1} \cdot \frac{6n+3}{n-2}$

Non-permissible values:

$n = -\frac{1}{2}$ and $n = 2$

$\frac{n^2-4}{2n+1} \cdot \frac{6n+3}{n-2}$

$= \frac{(n-2)(n+2)}{2n+1} \cdot \frac{3(2n+1)}{n-2}$

$= 3(n+2), n \neq -\frac{1}{2}, 2$

c) $\frac{x+5}{x-4} \div \frac{x^2-25}{3x-12}$

$= \frac{x+5}{x-4} \div \frac{(x-5)(x+5)}{3(x-4)}$

Non-permissible values:

$x = -5$, $x = 4$, and $x = 5$

$= \frac{x+5}{x-4} \cdot \frac{3(x-4)}{(x-5)(x+5)}$

$= \frac{3}{x-5}, x \neq -5, 4, 5$

d) $\frac{(b+2)^2}{3b^2-3} \div \frac{2b+4}{1-b}$

$= \frac{(b+2)^2}{3(b^2-1)} \div \frac{2(b+2)}{1-b}$

$= \frac{(b+2)^2}{3(b-1)(b+1)} \div \frac{2(b+2)}{-(b-1)}$

Non-permissible values:

$b = -2$, $b = -1$, and $b = 1$

$= \frac{(b+2)^2}{3(b-1)(b+1)} \cdot \frac{-(b-1)}{2(b+2)}$

$= -\frac{b+2}{6(b+1)}, b \neq -2, -1, 1$

8. Simplify each expression.

a) $\frac{x^2-x-6}{x+4} \cdot \frac{x^2-16}{x^2+2x}$

$= \frac{(x-3)(x+2)}{x+4} \cdot \frac{(x-4)(x+4)}{x(x+2)}$

Non-permissible values:

$x = -4, -2, 0$

$= \frac{(x-3)(x+2)}{x+4} \cdot \frac{(x-4)(x+4)}{x(x+2)}$

$= \frac{(x-3)(x-4)}{x}, x \neq -4, -2, 0$

b) $\frac{2x^2-x-1}{x^2+2x-3} \cdot \frac{4x^2+28x+48}{2x^2-13x-7}$

$= \frac{(2x+1)(x-1)}{(x+3)(x-1)} \cdot \frac{4(x^2+7x+12)}{(2x+1)(x-7)}$

$= \frac{(2x+1)(x-1)}{(x+3)(x-1)} \cdot \frac{4(x+3)(x+4)}{(2x+1)(x-7)}$

Non-permissible values:

$x = -3, -\frac{1}{2}, 1, 7$

$= \frac{(2x+1)(x-1)}{(x+3)(x-1)} \cdot \frac{4(x+3)(x+4)}{(2x+1)(x-7)}$

$= \frac{4(x+4)}{x-7}, x \neq -3, -\frac{1}{2}, 1, 7$

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c) $\frac{m^2 - m - 12}{m + 3} \div \frac{3m - 12}{m^2 - 9}$

$= \frac{(m - 4)(m + 3)}{m + 3} \div \frac{3(m - 4)}{(m - 3)(m + 3)}$

Non-permissible values:

$m = -3, 3, 4$

$= \frac{\cancel{(m - 4)}\ \cancel{(m + 3)}}{\cancel{m + 3}} \cdot \frac{(m - 3)(m + 3)}{3\ \cancel{(m - 4)}}$

$= \frac{(m - 3)(m + 3)}{3}, m \neq -3, 3, 4$

d) $\frac{2s^2 - 7s - 4}{6s^2 - 5s - 6} \div \frac{4s^2 + 4s + 1}{12s + 8}$

$= \frac{(2s + 1)(s - 4)}{(3s + 2)(2s - 3)} \div \frac{(2s + 1)(2s + 1)}{4(3s + 2)}$

Non-permissible values:

$s = -\frac{2}{3}, -\frac{1}{2}, \frac{3}{2}$

$= \frac{\cancel{(2s + 1)}\ (s - 4)}{\cancel{(3s + 2)}\ (2s - 3)} \cdot \frac{4\ \cancel{(3s + 2)}}{(2s + 1)\ \cancel{(2s + 1)}}$

$= \frac{4(s - 4)}{(2s - 3)(2s + 1)}, s \neq -\frac{2}{3}, -\frac{1}{2}, \frac{3}{2}$

9. Each rational expression is the product of two different rational expressions. What might the rational expressions have been? Describe the strategy you used to find out.

a) $\frac{3}{y}$

b) $\frac{x + 1}{3x}$

I wrote two polynomial expressions such that when I divided the numerators and denominators by their common factors, I was left with the given product. I then expanded where necessary.

Sample Response:

$\frac{3}{y} = \frac{3(2x)}{(5)} \cdot \frac{(5)}{(2x)y}$

$= \frac{6x}{5} \cdot \frac{5}{2xy}, x \neq 0, y \neq 0$

$\frac{x + 1}{3x} = \frac{(x + 1)(x + 2)}{3x(2)} \cdot \frac{(2)}{(x + 2)}$

$= \frac{x^2 + 3x + 2}{6x} \cdot \frac{2}{x + 2}, x \neq -2, 0$

10. Each rational expression is the quotient of two different rational expressions. What might the rational expressions have been? How do you know?

a) $\frac{3}{x - 5}$

b) $\frac{x - 9}{x + 8}$

I wrote a product of two polynomial expressions such that when I divided the numerators and denominators by their common factors, I was left with the given expression. I then expanded where necessary, inverted the second expression, then changed multiplication to division.

Sample Response:

$\frac{3}{x - 5} = \frac{3(5)}{(x - 5)(x + 4)} \cdot \frac{x + 4}{(5)}$

$= \frac{15}{(x - 5)(x + 4)} \div \frac{5}{x + 4}$,

$x \neq -4, 5$

$\frac{x - 9}{x + 8} = \frac{(x - 9)^2}{2(x + 8)} \cdot \frac{2}{(x - 9)}$

$= \frac{x^2 - 18x + 81}{2x + 16} \div \frac{x - 9}{2}$,

$x \neq -8, 9$

11. Simplify each expression.

a) $\frac{3x^3y}{18x^3} \cdot \frac{2y^2}{4xy^5} \cdot \frac{8x^3y^2}{12xy}$

Non-permissible values:
$x = 0, y = 0$

$= \frac{\cancel{3}\,\cancel{x^3}\,\cancel{y}}{\cancel{18}\,\cancel{x^3}} \cdot \frac{\cancel{2}\,\cancel{y^2}}{\cancel{4}\,\cancel{x}\,\cancel{y^5}} \cdot \frac{\cancel{8}\,\cancel{x^3}\,\cancel{y^2}}{\cancel{12}\,\cancel{x}\,\cancel{y}}$

$= \frac{x}{18y}, x \neq 0, y \neq 0$

b) $\frac{3ab}{4ab^3} \cdot \frac{8b^2}{27a^5b^3} \div \frac{2a^2b^2}{6ab^3}$

Non-permissible values:
$a = 0, b = 0$

$= \frac{\cancel{3}\,\cancel{a}\,\cancel{b}}{\cancel{4}\,\cancel{a}\,\cancel{b^3}} \cdot \frac{\cancel{8}\,\cancel{b^2}}{\cancel{27}\,a^5\cancel{b^2}} \cdot \frac{\cancel{6}\,\cancel{a}\,\cancel{b^3}}{\cancel{2}\,a^2\cancel{b^2}}$

$= \frac{2}{3a^6b^2}, a \neq 0, b \neq 0$

12. Simplify each expression.

a) $\frac{x + 3}{x^2 - 2x - 15} \cdot \frac{2x - 10}{2x^2 + 15x + 7} \div \frac{2x - 4}{x^2 + 5x - 14}$

$= \frac{x + 3}{(x - 5)(x + 3)} \cdot \frac{2(x - 5)}{(2x + 1)(x + 7)} \div \frac{2(x - 2)}{(x + 7)(x - 2)}$

Non-permissible values: $x = -7, -3, -\frac{1}{2}, 2, 5$

$= \frac{\cancel{x + 3}}{\cancel{(x - 5)}\,\cancel{(x + 3)}} \cdot \frac{\cancel{2}\,\cancel{(x - 5)}}{(2x + 1)\,\cancel{(x + 7)}} \cdot \frac{\cancel{(x + 7)}\,\cancel{(x - 2)}}{\cancel{2}\,\cancel{(x - 2)}}$

$= \frac{1}{2x + 1}, x \neq -7, -3, -\frac{1}{2}, 2, 5$

b) $\frac{x^2 - x - 56}{x + 8} \div \frac{x^2 + 14x + 49}{x^2 - 6x - 16} \cdot \frac{3x + 24}{x^2 - 16x + 64}$

$= \frac{(x - 8)(x + 7)}{x + 8} \div \frac{(x + 7)(x + 7)}{(x - 8)(x + 2)} \cdot \frac{3(x + 8)}{(x - 8)(x - 8)}$

Non-permissible values: $x = -8, -7, -2, 8$

$= \frac{\cancel{(x - 8)}\,\cancel{(x + 7)}}{\cancel{x + 8}} \cdot \frac{\cancel{(x - 8)}\,(x + 2)}{\cancel{(x + 7)}\,(x + 7)} \cdot \frac{3\,\cancel{(x + 8)}}{\cancel{(x - 8)}\,\cancel{(x - 8)}}$

$= \frac{3(x + 2)}{x + 7}, x \neq -8, -7, -2, 8$

c) $\frac{x + 4y}{x - 5y} \cdot \frac{x^2 - 25y^2}{x^2 - 16y^2} \div \frac{x + 5y}{x - 4y}$

$= \frac{x + 4y}{x - 5y} \cdot \frac{(x - 5y)(x + 5y)}{(x - 4y)(x + 4y)} \div \frac{x + 5y}{x - 4y}$

Non-permissible values: $x = -5y, -4y, 4y, 5y$

$= \frac{\cancel{x + 4y}}{\cancel{x - 5y}} \cdot \frac{\cancel{(x - 5y)}\,\cancel{(x + 5y)}}{\cancel{(x - 4y)}\,\cancel{(x + 4y)}} \cdot \frac{\cancel{x - 4y}}{\cancel{x + 5y}}$

$= 1, x \neq -5y, -4y, 4y, 5y$

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d) $\dfrac{(5c + 6d)^2}{3c - 4d} \cdot \dfrac{9c^2 - 16d^2}{25c^2 - 36d^2} \div \dfrac{3c + 4d}{5c - 6d}$

$= \dfrac{(5c + 6d)^2}{3c - 4d} \cdot \dfrac{(3c - 4d)(3c + 4d)}{(5c - 6d)(5c + 6d)} \div \dfrac{3c + 4d}{5c - 6d}$

Non-permissible values: $c = -\dfrac{4d}{3}, -\dfrac{6d}{5}, \dfrac{6d}{5}, \dfrac{4d}{3}$

$= \dfrac{(5c + 6d)^{\cancel{2}}}{\cancel{3c - 4d}} \cdot \dfrac{\cancel{(3c - 4d)}\ \cancel{(3c + 4d)}}{\cancel{(5c - 6d)}\ \cancel{(5c + 6d)}} \cdot \dfrac{\cancel{5c - 6d}}{\cancel{3c + 4d}}$

$= 5c + 6d, c \neq -\dfrac{4d}{3}, -\dfrac{6d}{5}, \dfrac{6d}{5}, \dfrac{4d}{3}$

13. Simplify each expression.

$\dfrac{x^2 - 2x - 8}{x + 5} \div \dfrac{x^2 - x - 6}{x^2 + 2x - 15}$

$\dfrac{x^2 - x - 6}{x^2 + 2x - 15} \div \dfrac{x^2 - 2x - 8}{x + 5}$

Compare the solutions.
What do you notice?

$\dfrac{x^2 - 2x - 8}{x + 5} \div \dfrac{x^2 - x - 6}{x^2 + 2x - 15}$

$= \dfrac{(x - 4)(x + 2)}{x + 5} \div \dfrac{(x - 3)(x + 2)}{(x + 5)(x - 3)}$

Non-permissible values:
$x = -5, -2, 3$

$= \dfrac{(x - 4)\ \cancel{(x + 2)}}{\cancel{x + 5}} \cdot \dfrac{\cancel{(x + 5)}\ \cancel{(x - 3)}}{\cancel{(x - 3)}\ \cancel{(x + 2)}}$

$= x - 4, x \neq -5, -2, 3$

The solutions are reciprocals.

$\dfrac{x^2 - x - 6}{x^2 + 2x - 15} \div \dfrac{x^2 - 2x - 8}{x + 5}$

$= \dfrac{(x - 3)(x + 2)}{(x + 5)(x - 3)} \div \dfrac{(x - 4)(x + 2)}{x + 5}$

Non-permissible values:
$x = -5, -2, 3, 4$

$= \dfrac{\cancel{(x - 3)}\ \cancel{(x + 2)}}{\cancel{(x + 5)}\ \cancel{(x - 3)}} \cdot \dfrac{x + 5}{(x - 4)\ \cancel{(x + 2)}}$

$= \dfrac{1}{x - 4}, x \neq -5, -2, 3, 4$

C

14. a) Write a polynomial A so that the expression below simplifies to -1.

$\dfrac{3n^2 + 2n - 8}{n^2 + 4n + 4} \cdot \dfrac{A}{3n^2 - n - 4}, n \neq -2, -1, \dfrac{4}{3}$

$= \dfrac{\cancel{(3n - 4)}\ \cancel{(n + 2)}}{\cancel{(n + 2)}\ (n + 2)} \cdot \dfrac{A}{\cancel{(3n - 4)}\ (n + 1)}$

$= \dfrac{A}{(n + 2)(n + 1)}$

Set the expression equal to -1.

$-1 = \dfrac{A}{(n + 2)(n + 1)}$

$A = -(n + 2)(n + 1)$

$A = -(n^2 + 3n + 2)$

$A = -n^2 - 3n - 2$

b) Write a polynomial B so that the expression below simplifies to $\frac{1}{5}$.

$$\frac{x^2 + 6x + 9}{5x^2 + 15x} \div \frac{3x^2 + 11x + 6}{B}, x \neq -3, 0$$

$$= \frac{(x + 3)(x + 3)}{5x(x + 3)} \div \frac{(x + 3)(3x + 2)}{B}$$

$$= \frac{\cancel{(x + 3)}\ \cancel{(x + 3)}}{5x\ \cancel{(x + 3)}} \cdot \frac{B}{\cancel{(x + 3)}\ (3x + 2)}$$

$$= \frac{B}{5x(3x + 2)}$$

Set the expression equal to $\frac{1}{5}$.

$$\frac{1}{5} = \frac{B}{5x(3x + 2)}$$

$$B = \frac{5x(3x + 2)}{5}$$

$$B = x(3x + 2)$$

$$B = 3x^2 + 2x$$

Since B is in the denominator, $x \neq -\frac{2}{3}, 0$

15. Simplify: $\dfrac{1 - \frac{2}{a}}{1 - \frac{4}{a^2}}$

$$= \frac{\frac{a - 2}{a}}{\frac{a^2 - 4}{a^2}}$$

$$= \frac{a - 2}{a} \cdot \frac{a^2}{a^2 - 4}$$

$$= \frac{\cancel{a - 2}}{\cancel{a}} \cdot \frac{a^{\cancel{2}}}{\cancel{(a - 2)}\ (a + 2)}$$ Non-permissible values: $a = -2, 0, 2$

$$= \frac{a}{a + 2}, a \neq -2, 0, 2$$

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