Lesson 7.3 Exercises, pages 553–559

A

3. Simplify.

a) $\frac{5}{a} + \frac{3}{a}$

$= \frac{8}{a}, a \neq 0$

b) $\frac{1}{9x} + \frac{4}{9x}$

$= \frac{5}{9x}, x \neq 0$

c) $\frac{7e}{e + 2} - \frac{3e}{e + 2}$

$= \frac{4e}{e + 2}, e \neq -2$

d) $\frac{3h}{h^2 + 3h + 4} - \frac{8h}{h^2 + 3h + 4}$

$= \frac{-5h}{h^2 + 3h + 4}$

4. **a)** Determine the least common multiple of the expressions in each pair.

i) $4d, 5d$

$4d = 2 \cdot 2 \cdot d$
$5d = 5 \cdot d$
Least common multiple:
$2 \cdot 2 \cdot 5 \cdot d = 20d$

ii) $6e^2, 7e$

$6e^2 = 2 \cdot 3 \cdot e \cdot e$
$7e = 7 \cdot e$
Least common multiple:
$2 \cdot 3 \cdot 7 \cdot e \cdot e = 42e^2$

b) Use your results from part a to write each pair of expressions with a common denominator.

i) $\frac{3}{4d}, \frac{-2}{5d}$

Common denominator: $20d$
$\frac{3}{4d} \cdot \frac{5}{5} = \frac{15}{20d}$
$\frac{-2}{5d} \cdot \frac{4}{4} = \frac{-8}{20d}$

ii) $\frac{5}{6e^2}, \frac{6}{7e}$

Common denominator: $42e^2$
$\frac{5}{6e^2} \cdot \frac{7}{7} = \frac{35}{42e^2}$
$\frac{6}{7e} \cdot \frac{6e}{6e} = \frac{36e}{42e^2}$

c) Simplify.

i) $\frac{3}{4d} + \frac{-2}{5d}$

Non-permissible value: $d = 0$
$= \frac{15}{20d} + \frac{-8}{20d}$
$= \frac{7}{20d}, d \neq 0$

ii) $\frac{5}{6e^2} - \frac{6}{7e}$

Non-permissible value: $e = 0$
$= \frac{35}{42e^2} - \frac{36e}{42e^2}$
$= \frac{35 - 36e}{42e^2}, e \neq 0$

B

5. Simplify.

a) $\frac{4}{a} - \frac{2}{3}$

Non-permissible value: $a = 0$
Common denominator: $3a$
$= \frac{4}{a} \cdot \frac{3}{3} - \frac{2}{3} \cdot \frac{a}{a}$
$= \frac{12}{3a} - \frac{2a}{3a}$
$= \frac{12 - 2a}{3a}, a \neq 0$

b) $\frac{4}{3x} + \frac{1}{5x}$

Non-permissible value: $x = 0$
Common denominator: $15x$
$= \frac{4}{3x} \cdot \frac{5}{5} + \frac{1}{5x} \cdot \frac{3}{3}$
$= \frac{20}{15x} + \frac{3}{15x}$
$= \frac{23}{15x}, x \neq 0$

6. Simplify.

a) $\frac{1}{3a} + \frac{2a}{5a^2}$

Non-permissible value: $a = 0$
Common denominator: $15a^2$
$= \frac{1}{3a} \cdot \frac{5a}{5a} + \frac{2a}{5a^2} \cdot \frac{3}{3}$
$= \frac{5a}{15a^2} + \frac{6a}{15a^2}$
$= \frac{11\cancel{a}}{15a^{\cancel{2}}}$
$= \frac{11}{15a}, a \neq 0$

b) $4 - \frac{1}{x}$

Non-permissible value: $x = 0$
Common denominator: x
$= \frac{4}{1} \cdot \frac{x}{x} - \frac{1}{x}$
$= \frac{4x}{x} - \frac{1}{x}$
$= \frac{4x - 1}{x}, x \neq 0$

7. Simplify.

a) $\frac{2}{xz^2} - \frac{5}{x^2z}$

Non-permissible values:
$x = 0, z = 0$
Common denominator: x^2z^2

$= \frac{2}{xz^2} \cdot \frac{x}{x} - \frac{5}{x^2z} \cdot \frac{z}{z}$

$= \frac{2x}{x^2z^2} - \frac{5z}{x^2z^2}$

$= \frac{2x - 5z}{x^2z^2}, x \neq 0, z \neq 0$

b) $\frac{3}{r^2s^2} + \frac{2}{rs^3}$

Non-permissible values:
$r = 0, s = 0$
Common denominator: r^2s^3

$= \frac{3}{r^2s^2} \cdot \frac{s}{s} + \frac{2}{rs^3} \cdot \frac{r}{r}$

$= \frac{3s}{r^2s^3} + \frac{2r}{r^2s^3}$

$= \frac{3s + 2r}{r^2s^3}, r \neq 0, s \neq 0$

c) $\frac{1}{x} + \frac{7}{6y}$

Non-permissible values:
$x = 0, y = 0$
Common denominator: $6xy$

$= \frac{1}{x} \cdot \frac{6y}{6y} + \frac{7}{6y} \cdot \frac{x}{x}$

$= \frac{6y}{6xy} + \frac{7x}{6xy}$

$= \frac{6y + 7x}{6xy}, x \neq 0, y \neq 0$

d) $\frac{2}{3e^5f^2} + \frac{3}{5ef^3}$

Non-permissible values:
$e = 0, f = 0$
Common denominator: $15e^5f^3$

$= \frac{2}{3e^5f^2} \cdot \frac{5f}{5f} + \frac{3}{5ef^3} \cdot \frac{3e^4}{3e^4}$

$= \frac{10f}{15e^5f^3} + \frac{9e^4}{15e^5f^3}$

$= \frac{10f + 9e^4}{15e^5f^3}, e \neq 0, f \neq 0$

8. Simplify.

a) $\frac{2x - 3}{x} + \frac{x - 1}{3x}$

Non-permissible value: $x = 0$
Common denominator: $3x$

$= \frac{2x - 3}{x} \cdot \frac{3}{3} + \frac{x - 1}{3x}$

$= \frac{3(2x - 3)}{3x} + \frac{x - 1}{3x}$

$= \frac{6x - 9 + x - 1}{3x}$

$= \frac{7x - 10}{3x}, x \neq 0$

b) $\frac{2p + 4}{2p} + \frac{3p + 1}{3p^2}$

Non-permissible value: $p = 0$
Common denominator: $6p^2$

$= \frac{2p + 4}{2p} \cdot \frac{3p}{3p} + \frac{3p + 1}{3p^2} \cdot \frac{2}{2}$

$= \frac{3p(2p + 4)}{6p^2} + \frac{2(3p + 1)}{6p^2}$

$= \frac{6p^2 + 12p + 6p + 2}{6p^2}$

$= \frac{6p^2 + 18p + 2}{6p^2}$

$= \frac{\cancel{2}(3p^2 + 9p + 1)}{3\cancel{6}p^2}$

$= \frac{3p^2 + 9p + 1}{3p^2}, p \neq 0$

c) $\frac{3t-4}{t} - \frac{2t-7}{8}$

Non-permissible value: $t = 0$
Common denominator: $8t$

$= \frac{3t-4}{t} \cdot \frac{8}{8} - \frac{2t-7}{8} \cdot \frac{t}{t}$

$= \frac{8(3t-4)}{8t} - \frac{t(2t-7)}{8t}$

$= \frac{24t - 32 - 2t^2 + 7t}{8t}$

$= \frac{-2t^2 + 31t - 32}{8t}, t \neq 0$

d) $\frac{2q-3}{5q^2} - \frac{1-q}{4q^3}$

Non-permissible value: $q = 0$
Common denominator: $20q^3$

$= \frac{2q-3}{5q^2} \cdot \frac{4q}{4q} - \frac{1-q}{4q^3} \cdot \frac{5}{5}$

$= \frac{4q(2q-3)}{20q^3} - \frac{5(1-q)}{20q^3}$

$= \frac{8q^2 - 12q - 5 + 5q}{20q^3}$

$= \frac{8q^2 - 7q - 5}{20q^3}, q \neq 0$

9. Simplify.

a) $\frac{a+b}{5a} - \frac{b-a}{15b}$

Non-permissible values:
$a = 0, b = 0$
Common denominator: $15ab$

$= \frac{a+b}{5a} \cdot \frac{3b}{3b} - \frac{b-a}{15b} \cdot \frac{a}{a}$

$= \frac{3b(a+b)}{15ab} - \frac{a(b-a)}{15ab}$

$= \frac{3ab + 3b^2 - ab + a^2}{15ab}$

$= \frac{3b^2 + 2ab + a^2}{15ab}, a \neq 0, b \neq 0$

b) $\frac{2x-y}{x^2y} + \frac{x+y}{xy^2}$

Non-permissible values:
$x = 0, y = 0$
Common denominator: x^2y^2

$= \frac{2x-y}{x^2y} \cdot \frac{y}{y} + \frac{x+y}{xy^2} \cdot \frac{x}{x}$

$= \frac{y(2x-y)}{x^2y^2} + \frac{x(x+y)}{x^2y^2}$

$= \frac{2xy - y^2 + x^2 + xy}{x^2y^2}$

$= \frac{x^2 + 3xy - y^2}{x^2y^2}, x \neq 0, y \neq 0$

c) $\frac{11j-7}{6j^2} - \frac{k+2}{7k}$

Non-permissible values:
$j = 0, k = 0$
Common denominator: $42j^2k$

$= \frac{11j-7}{6j^2} \cdot \frac{7k}{7k} - \frac{k+2}{7k} \cdot \frac{6j^2}{6j^2}$

$= \frac{7k(11j-7)}{42j^2k} - \frac{6j^2(k+2)}{42j^2k}$

$= \frac{77jk - 49k - 6j^2k - 12j^2}{42j^2k},$

$j \neq 0, k \neq 0$

d) $\frac{b+1}{7bc} - \frac{b^2-1}{8b^3}$

Non-permissible values:
$b = 0, c = 0$
Common denominator: $56b^3c$

$= \frac{b+1}{7bc} \cdot \frac{8b^2}{8b^2} - \frac{b^2-1}{8b^3} \cdot \frac{7c}{7c}$

$= \frac{8b^2(b+1)}{56b^3c} - \frac{7c(b^2-1)}{56b^3c}$

$= \frac{8b^3 + 8b^2 - 7b^2c + 7c}{56b^3c},$

$b \neq 0, c \neq 0$

10. Here is a student's solution for subtracting rational expressions. Identify the error in the solution. Write a correct solution.

$$\frac{3x+6}{2x^2}-\frac{x-4}{3x}=\frac{3x+6}{2x^2}\cdot\frac{3}{3}-\frac{x-4}{3x}\cdot\frac{2x}{2x}$$
$$=\frac{3(3x+6)}{6x^2}-\frac{2x(x-4)}{6x^2}$$
$$=\frac{9x+18-2x^2-8x}{6x^2}$$
$$=\frac{-2x^2+x+18}{6x^2},\ x\neq 0$$

There is an error in line 3: $-2x(-4)$ should be $8x$, not $-8x$.
Correct solution:

$$\frac{3x+6}{2x^2}-\frac{x-4}{3x}=\frac{3x+6}{2x^2}\cdot\frac{3}{3}-\frac{x-4}{3x}\cdot\frac{2x}{2x}$$
$$=\frac{3(3x+6)}{6x^2}-\frac{2x(x-4)}{6x^2}$$
$$=\frac{9x+18-2x^2+8x}{6x^2}$$
$$=\frac{-2x^2+17x+18}{6x^2},\ x\neq 0$$

11. Simplify.

a) $\frac{6}{3x}-\frac{2}{4x}+\frac{3}{6x}$

Non-permissible value: $x=0$
Common denominator: $12x$

$$=\frac{6}{3x}\cdot\frac{4}{4}-\frac{2}{4x}\cdot\frac{3}{3}+\frac{3}{6x}\cdot\frac{2}{2}$$
$$=\frac{24}{12x}-\frac{6}{12x}+\frac{6}{12x}$$
$$=\frac{24}{12x}$$
$$=\frac{2}{x},\ x\neq 0$$

b) $\frac{2}{u}-\frac{3}{u^2}+\frac{1}{2u}$

Non-permissible value: $u=0$
Common denominator: $2u^2$

$$=\frac{2}{u}\cdot\frac{2u}{2u}-\frac{3}{u^2}\cdot\frac{2}{2}+\frac{1}{2u}\cdot\frac{u}{u}$$
$$=\frac{4u}{2u^2}-\frac{6}{2u^2}+\frac{u}{2u^2}$$
$$=\frac{4u-6+u}{2u^2}$$
$$=\frac{5u-6}{2u^2},\ u\neq 0$$

c) $\frac{-10}{4n}+\frac{5}{3}-\frac{6}{m}$

Non-permissible values:
$n=0$ and $m=0$
Common denominator: $12nm$

$$=\frac{-10}{4n}\cdot\frac{3m}{3m}+\frac{5}{3}\cdot\frac{4mn}{4mn}-\frac{6}{m}\cdot\frac{12n}{12n}$$
$$=\frac{-30m}{12mn}+\frac{20mn}{12mn}-\frac{72n}{12mn}$$
$$=\frac{2(-15m+10mn-36n)}{12mn}$$
$$=\frac{-15m+10mn-36n}{6mn},$$
$$m\neq 0, n\neq 0$$

d) $\frac{4j-5}{3j}+\frac{2j+1}{5j^2}-\frac{7-3j}{2}$

Non-permissible value: $j=0$
Common denominator: $30j^2$

$$=\frac{(4j-5)}{3j}\cdot\frac{10j}{10j}+\frac{(2j+1)}{5j^2}\cdot\frac{6}{6}-\frac{(7-3j)}{2}\cdot\frac{15j^2}{15j^2}$$
$$=\frac{10j(4j-5)}{30j^2}+\frac{6(2j+1)}{30j^2}-\frac{15j^2(7-3j)}{30j^2}$$
$$=\frac{40j^2-50j+12j+6-105j^2+45j^3}{30j^2}$$
$$=\frac{45j^3-65j^2-38j+6}{30j^2},\ j\neq 0$$

12. Simplify.

a) $\frac{x^2 - 3}{2x} + x - 2$

Non-permissible value: $x = 0$

Common denominator: $2x$

$= \frac{x^2 - 3}{2x} + \frac{x}{1} \cdot \frac{2x}{2x} - \frac{2}{1} \cdot \frac{2x}{2x}$

$= \frac{x^2 - 3}{2x} + \frac{2x^2}{2x} - \frac{4x}{2x}$

$= \frac{3x^2 - 4x - 3}{2x}, x \neq 0$

b) $4w + \frac{1}{v} - \frac{2w - 3}{w^2}$

Non-permissible values: $v = 0$, $w = 0$

Common denominator: vw^2

$= \frac{4w}{1} \cdot \frac{vw^2}{vw^2} + \frac{1}{v} \cdot \frac{w^2}{w^2} - \frac{(2w - 3)}{w^2} \cdot \frac{v}{v}$

$= \frac{4vw^3}{vw^2} + \frac{w^2}{vw^2} - \frac{v(2w - 3)}{vw^2}$

$= \frac{4vw^3 + w^2 - 2vw + 3v}{vw^2}, v \neq 0, w \neq 0$

c) $\frac{x + y}{y} + \frac{x - y}{x} - 4xy$

Non-permissible values:
$x = 0$ and $y = 0$

Common denominator: xy

$= \frac{(x + y)}{y} \cdot \frac{x}{x}$

$+ \frac{(x - y)}{x} \cdot \frac{y}{y} - \frac{4xy}{1} \cdot \frac{xy}{xy}$

$= \frac{x(x + y)}{xy} + \frac{y(x - y)}{xy} - \frac{4x^2y^2}{xy}$

$= \frac{x^2 + xy + xy - y^2 - 4x^2y^2}{xy}$

$= \frac{x^2 + 2xy - y^2 - 4x^2y^2}{xy}$,

$x \neq 0, y \neq 0$

d) $\frac{-2 + t}{6t} + 3 - \frac{2t + 3}{5}$

Non-permissible value:
$t = 0$

Common denominator: $30t$

$= \frac{(-2 + t)}{6t} \cdot \frac{5}{5} + \frac{3}{1} \cdot \frac{30t}{30t} - \frac{(2t + 3)}{5} \cdot \frac{6t}{6t}$

$= \frac{5(-2 + t) + 90t - 6t(2t + 3)}{30t}$

$= \frac{-10 + 5t + 90t - 12t^2 - 18t}{30t}$

$= \frac{-12t^2 + 77t - 10}{30t}, t \neq 0$

13. Simplify.

a) $\frac{3}{2a} + \frac{5a^3}{3ab^3} \cdot \frac{12b^2}{10a^2}$

Non-permissible values:
$a = 0, b = 0$

$= \frac{3}{2a} + \frac{\cancel{5}\,\cancel{a^3}}{\cancel{3}\,\cancel{a}\,\cancel{b^3}} \cdot \frac{\cancel{12}\,\cancel{b^2}}{\cancel{10}\,\cancel{a^2}}$

$= \frac{3}{2a} + \frac{2}{b}$

Common denominator: $2ab$

$= \frac{3}{2a} \cdot \frac{b}{b} + \frac{2}{b} \cdot \frac{2a}{2a}$

$= \frac{3b + 4a}{2ab}, a \neq 0, b \neq 0$

b) $\frac{6x}{2(x + 3)} \div \frac{4x^2}{5(x + 3)} - \frac{x}{2}$

Non-permissible values:
$x = -3, x = 0$

$= \frac{\cancel{6}\,x}{\cancel{2}\,\cancel{(x + 3)}} \cdot \frac{5\,\cancel{(x + 3)}}{4x^{\cancel{2}}} - \frac{x}{2}$

$= \frac{15}{4x} - \frac{x}{2}$

Common denominator: $4x$

$= \frac{15}{4x} - \frac{x}{2} \cdot \frac{2x}{2x}$

$= \frac{15 - 2x^2}{4x}, x \neq -3, 0$

C

14. Is the statement $\frac{a}{x} + \frac{b}{y} = \frac{ay + bx}{xy}$, for $x, y \neq 0$, sometimes true, always true, or never true? Justify your answer.

Non-permissible values: $x = 0$, $y = 0$

$$\frac{a}{x} + \frac{b}{y} = \frac{a}{x} \cdot \frac{y}{y} + \frac{b}{y} \cdot \frac{x}{x}$$
$$= \frac{ay}{xy} + \frac{bx}{xy}$$
$$= \frac{ay + bx}{xy}, x \neq 0, y \neq 0$$

Always true

15. Determine the sum of the first 6 terms of each series.

a) $\frac{1}{x} + \frac{1}{2x} + \frac{1}{3x} + \frac{1}{4x} + \ldots$

$\frac{1}{x} + \frac{1}{2x} + \frac{1}{3x} + \frac{1}{4x} + \frac{1}{5x} + \frac{1}{6x}$

A common denominator is $120x$.

A non-permissible value is $x = 0$.

$$= \frac{1}{x} \cdot \frac{120}{120} + \frac{1}{2x} \cdot \frac{60}{60} + \frac{1}{3x} \cdot \frac{40}{40} + \frac{1}{4x} \cdot \frac{30}{30} + \frac{1}{5x} \cdot \frac{24}{24} + \frac{1}{6x} \cdot \frac{20}{20}$$
$$= \frac{120 + 60 + 40 + 30 + 24 + 20}{120x}$$
$$= \frac{294}{120x}, \text{ or } \frac{147}{60x}, x \neq 0$$

b) $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \ldots$

$\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5} + \frac{1}{x^6}$

A common denominator is x^6.

A non-permissible value is $x = 0$.

$$= \frac{1}{x} \cdot \frac{x^5}{x^5} + \frac{1}{x^2} \cdot \frac{x^4}{x^4} + \frac{1}{x^3} \cdot \frac{x^3}{x^3} + \frac{1}{x^4} \cdot \frac{x^2}{x^2} + \frac{1}{x^5} \cdot \frac{x}{x} + \frac{1}{x^6}$$
$$= \frac{x^5 + x^4 + x^3 + x^2 + x + 1}{x^6}, x \neq 0$$