Lesson 7.4 Exercises, pages 566–573

A

3. Simplify.

a) $\frac{4x}{x + 5} + \frac{6x}{x + 5}$

$= \frac{10x}{x + 5}, x \neq -5$

b) $\frac{n + 1}{n - 2} + \frac{n - 4}{n - 2}$

$= \frac{n + 1 + n - 4}{n - 2}$

$= \frac{2n - 3}{n - 2}, n \neq 2$

c) $\frac{2b}{3b^2 + 1} - \frac{3b}{3b^2 + 1}$

$= \frac{-b}{3b^2 + 1}$

d) $\frac{2c - 9}{c^2 + 2c + 1} - \frac{4c - 9}{c^2 + 2c + 1}$

$= \frac{2c - 9 - 4c + 9}{c^2 + 2c + 1}$

$= \frac{-2c}{(c + 1)^2}, c \neq -1$

4. a) Determine a common denominator for each pair of rational expressions, then write the expressions with the common denominator.

i) $\frac{1}{m + 3}, \frac{5}{m - 4}$

Common denominator: $(m + 3)(m - 4)$

$\frac{1}{(m + 3)} \cdot \frac{(m - 4)}{(m - 4)} = \frac{m - 4}{(m + 3)(m - 4)}$

$\frac{5}{(m - 4)} \cdot \frac{(m + 3)}{(m + 3)} = \frac{5m + 15}{(m + 3)(m - 4)}$

ii) $\frac{2}{n - 1}, \frac{3}{(n - 1)^2}$

Common denominator: $(n - 1)^2$

$\frac{2}{(n - 1)} \cdot \frac{(n - 1)}{(n - 1)} = \frac{2n - 2}{(n - 1)^2}$

$\frac{3}{(n - 1)^2}$

b) Simplify.

i) $\frac{1}{m + 3} + \frac{5}{m - 4}$

$= \frac{m - 4}{(m + 3)(m - 4)}$

$+ \frac{5m + 15}{(m + 3)(m - 4)}$

$= \frac{6m + 11}{(m + 3)(m - 4)}, m \neq -3, 4$

ii) $\frac{2}{n - 1} - \frac{3}{(n - 1)^2}$

$= \frac{2n - 2}{(n - 1)^2} - \frac{3}{(n - 1)^2}$

$= \frac{2n - 5}{(n - 1)^2}, n \neq 1$

B

5. Simplify.

a) $\frac{1}{x - 2} - \frac{2}{x + 2}$

Common denominator:
$(x - 2)(x + 2)$

$= \frac{1}{(x - 2)} \cdot \frac{(x + 2)}{(x + 2)}$

$- \frac{2}{(x + 2)} \cdot \frac{(x - 2)}{(x - 2)}$

$= \frac{x + 2}{(x - 2)(x + 2)} - \frac{2x - 4}{(x - 2)(x + 2)}$

$= \frac{-x + 6}{(x - 2)(x + 2)}, x \neq -2, 2$

b) $\frac{6}{a - 3} + \frac{2}{a + 7}$

Common denominator:
$(a - 3)(a + 7)$

$= \frac{6}{(a - 3)} \cdot \frac{(a + 7)}{(a + 7)}$

$+ \frac{2}{(a + 7)} \cdot \frac{(a - 3)}{(a - 3)}$

$= \frac{6a + 42}{(a - 3)(a + 7)} + \frac{2a - 6}{(a - 3)(a + 7)}$

$= \frac{8a + 36}{(a - 3)(a + 7)}$

$= \frac{4(2a + 9)}{(a - 3)(a + 7)}, a \neq -7, 3$

c) $\frac{7}{b + 9} - \frac{4}{b - 2}$

Common denominator:
$(b + 9)(b - 2)$

$= \frac{7}{(b + 9)} \cdot \frac{(b - 2)}{(b - 2)}$

$- \frac{4}{(b - 2)} \cdot \frac{(b + 9)}{(b + 9)}$

$= \frac{7b - 14}{(b + 9)(b - 2)} - \frac{4b + 36}{(b + 9)(b - 2)}$

$= \frac{3b - 50}{(b + 9)(b - 2)}, b \neq -9, 2$

d) $\frac{-5}{w} + \frac{2}{w - 4}$

Common denominator:
$w(w - 4)$

$= \frac{-5}{w} \cdot \frac{(w - 4)}{(w - 4)} + \frac{2}{(w - 4)} \cdot \frac{w}{w}$

$= \frac{-5w + 20}{w(w - 4)} + \frac{2w}{w(w - 4)}$

$= \frac{-3w + 20}{w(w - 4)}, w \neq 0, 4$

6. Simplify.

a) $\frac{6}{r-4} + \frac{r+5}{4-r}$

$= \frac{6}{r-4} + \frac{r+5}{-(r-4)}$

$= \frac{6}{r-4} - \frac{r+5}{r-4}$

$= \frac{-r+1}{r-4}, r \neq 4$

b) $\frac{t+3}{t-4} - \frac{t-2}{t-5}$

Common denominator:
$(t-4)(t-5)$

$= \frac{(t+3)}{(t-4)} \cdot \frac{(t-5)}{(t-5)} - \frac{(t-2)}{(t-5)} \cdot \frac{(t-4)}{(t-4)}$

$= \frac{t^2-2t-15}{(t-4)(t-5)} - \frac{t^2-6t+8}{(t-4)(t-5)}$

$= \frac{4t-23}{(t-4)(t-5)}, t \neq 4, 5$

7. Simplify.

a) $\frac{8}{6x+9} + \frac{3}{4x-4}$

$= \frac{8}{3(2x+3)} + \frac{3}{4(x-1)}$

Common denominator:
$12(2x+3)(x-1)$

$= \frac{8}{3(2x+3)} \cdot \frac{4(x-1)}{4(x-1)} + \frac{3}{4(x-1)} \cdot \frac{3(2x+3)}{3(2x+3)}$

$= \frac{32(x-1)}{12(2x+3)(x-1)} + \frac{9(2x+3)}{12(2x+3)(x-1)}$

$= \frac{32x-32+18x+27}{12(2x+3)(x-1)}$

$= \frac{50x-5}{12(2x+3)(x-1)}$

$= \frac{5(10x-1)}{12(2x+3)(x-1)}, x \neq -\frac{3}{2}, 1$

b) $\frac{10}{4k^2-4} - \frac{2}{5-5k}$

$= \frac{10}{4(k^2-1)} - \frac{2}{5(1-k)}$

$= \frac{10}{4(k-1)(k+1)} + \frac{2}{5(k-1)}$

Common denominator:
$20(k-1)(k+1)$

$= \frac{10}{4(k-1)(k+1)} \cdot \frac{5}{5} + \frac{2}{5(k-1)} \cdot \frac{4(k+1)}{4(k+1)}$

$= \frac{50}{20(k-1)(k+1)} + \frac{8k+8}{20(k-1)(k+1)}$

$= \frac{58+8k}{20(k-1)(k+1)}$

$= \frac{2(29+4k)}{20(k-1)(k+1)}$

$= \frac{29+4k}{10(k-1)(k+1)}, k \neq -1, 1$

8. Here is a student's solution for adding rational expressions. Identify the error in the solution. Write a correct solution.

$$\frac{2}{x+5} + \frac{7}{x+5} = \frac{2+7}{x+5+x+5}$$
$$= \frac{9}{2x+10},\ x \neq -5$$

The error is in line 1 of the solution. The student added the terms in the denominator. When two rational expressions have a common denominator, only the terms in the numerator are added. Correct solution:

$$\frac{2}{x+5} + \frac{7}{x+5} = \frac{2+7}{x+5}$$
$$= \frac{9}{x+5},\ x \neq -5$$

9. Write $\frac{2x-7}{x+5}$ as the difference of two rational expressions.
Compare your answers with those of a classmate.
Did you get the same answers? Explain.

To subtract 2 fractions with the same denominator, I subtract the numerators.

$$\frac{2x-7}{x+5} = \frac{3x-3}{x+5} - \frac{x+4}{x+5},\ x \neq -5$$

I did not get the same answers as my classmate. There are many possible answers. I can write many different differences of 2 rational expressions, each with denominator $x + 5$, so that the numerators have a difference of $2x - 7$.

10. Simplify.

a) $3 + \frac{5x^2+8}{x^2-2x-8}$

$$= \frac{3}{1} + \frac{5x^2+8}{(x-4)(x+2)}$$

Common denominator:
$(x-4)(x+2)$

$$= \frac{3}{1} \cdot \frac{(x-4)(x+2)}{(x-4)(x+2)} + \frac{5x^2+8}{(x-4)(x+2)}$$
$$= \frac{3(x^2-2x-8)}{(x-4)(x+2)} + \frac{5x^2+8}{(x-4)(x+2)}$$
$$= \frac{3x^2-6x-24+5x^2+8}{(x-4)(x+2)}$$
$$= \frac{8x^2-6x-16}{(x-4)(x+2)}$$
$$= \frac{2(4x^2-3x-8)}{(x-4)(x+2)},\ x \neq -2, 4$$

b) $\frac{b}{b^2+10b+24} + \frac{2b}{b^2+12b+32}$

$$= \frac{b}{(b+4)(b+6)} + \frac{2b}{(b+4)(b+8)}$$

Common denominator:
$(b+4)(b+6)(b+8)$

$$= \frac{b}{(b+4)(b+6)} \cdot \frac{(b+8)}{(b+8)} + \frac{2b}{(b+4)(b+8)} \cdot \frac{(b+6)}{(b+6)}$$
$$= \frac{b^2+8b}{(b+4)(b+6)(b+8)} + \frac{2b^2+12b}{(b+4)(b+6)(b+8)}$$
$$= \frac{3b^2+20b}{(b+4)(b+6)(b+8)}$$
$$= \frac{b(3b+20)}{(b+4)(b+6)(b+8)},$$
$b \neq -8, -6, -4$

c) $\dfrac{4u^2 - 20u}{u^2 + 2u - 35} - \dfrac{3u - 6}{3u^2 - 10u + 8}$

$= \dfrac{4u\cancel{(u - 5)}}{(u + 7)\cancel{(u - 5)}} - \dfrac{3\cancel{(u - 2)}}{(3u - 4)\cancel{(u - 2)}}$

$= \dfrac{4u}{(u + 7)} - \dfrac{3}{(3u - 4)}$

Common denominator:
$(u + 7)(3u - 4)$

$= \dfrac{4u}{(u + 7)} \cdot \dfrac{(3u - 4)}{(3u - 4)} - \dfrac{3}{(3u - 4)} \cdot \dfrac{(u + 7)}{(u + 7)}$

$= \dfrac{12u^2 - 16u}{(u + 7)(3u - 4)} - \dfrac{3u + 21}{(u + 7)(3u - 4)}$

$= \dfrac{12u^2 - 19u - 21}{(u + 7)(3u - 4)}, u \neq -7, \dfrac{4}{3}, 2, 5$

11. A cycling club rode a 50-km round-trip route. On the return trip, the cyclists had a tailwind so their average speed was 7 km/h greater than on the first half of the trip. Write an expression for the total time spent cycling in terms of the average speed for the first half of the trip.

Let x kilometres per hour represent the average speed for the first half of the trip. From the context, $x > 0$. Then, the average speed on the return trip is $(x + 7)$ km/h.

Each leg of the trip is $\dfrac{50 \text{ km}}{2}$, or 25 km

Cycling time on first half: $\dfrac{25}{x}$ h

Cycling time on return trip: $\dfrac{25}{x + 7}$ h

Total time cycling is: $\dfrac{25}{x} + \dfrac{25}{x + 7}$

A common denominator is: $x(x + 7)$

$$\frac{25}{x} + \frac{25}{x + 7} = \frac{25}{x} \cdot \frac{(x + 7)}{(x + 7)} + \frac{25}{(x + 7)} \cdot \frac{x}{x}$$

$$= \frac{25x + 175 + 25x}{x(x + 7)}$$

$$= \frac{50x + 175}{x(x + 7)}$$

$$= \frac{25(2x + 7)}{x(x + 7)}, x \neq -7, 0$$

So, the cycling time, in hours, is represented by the expression:

$\dfrac{25(2x + 7)}{x(x + 7)}, x > 0$

12. Simplify.

a) $\dfrac{x+2}{x^2+5x+6} - \dfrac{2+x}{4-x^2} + \dfrac{2-x}{x^2+x-6}$

$$= \frac{\cancel{x+2}}{\cancel{(x+2)}(x+3)} - \frac{\cancel{2+x}}{(2-x)\cancel{(2+x)}} + \frac{2-x}{(x+3)(x-2)}$$

$$= \frac{1}{x+3} - \frac{1}{-(x-2)} + \frac{-\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$$

$$= \frac{1}{x+3} + \frac{1}{x-2} + \frac{-1}{x+3}$$

$$= \frac{1}{x-2},\ x \neq -3, -2, 2$$

b) $\dfrac{x^2+3x+2}{x^2-1} + \dfrac{x^2+x-2}{x^2-x} - \dfrac{x^2-x-12}{x^2-3x-4}$

$$= \frac{\cancel{(x+1)}(x+2)}{(x-1)\cancel{(x+1)}} + \frac{(x+2)\cancel{(x-1)}}{x\cancel{(x-1)}} - \frac{\cancel{(x-4)}(x+3)}{\cancel{(x-4)}(x+1)}$$

$$= \frac{x+2}{x-1} + \frac{x+2}{x} - \frac{x+3}{x+1}$$

Common denominator:
$x(x-1)(x+1)$

$$= \frac{x+2}{x-1}\cdot\frac{x(x+1)}{x(x+1)} + \frac{x+2}{x}\cdot\frac{(x+1)(x-1)}{(x+1)(x-1)} - \frac{x+3}{x+1}\cdot\frac{x(x-1)}{x(x-1)}$$

$$= \frac{(x+2)(x^2+x)}{x(x-1)(x+1)} + \frac{(x+2)(x^2-1)}{x(x-1)(x+1)} - \frac{(x+3)(x^2-x)}{x(x-1)(x+1)}$$

$$= \frac{x^3+x^2+2x^2+2x+x^3-x+2x^2-2-x^3+x^2-3x^2+3x}{x(x-1)(x+1)}$$

$$= \frac{x^3+3x^2+4x-2}{x(x-1)(x+1)},\ x \neq -1, 0, 1, 4$$

13. Simplify.

a) $\dfrac{2x}{x+3} + \dfrac{3x}{2x+8} \div \dfrac{x^2}{3x+12}$

$$= \frac{2x}{x+3} + \frac{3\cancel{x}}{2\cancel{(x+4)}}\cdot\frac{3\cancel{(x+4)}}{x^{\cancel{2}}}$$

$$= \frac{2x}{x+3} + \frac{9}{2x}$$

Common denominator:
$2x(x+3)$

$$= \frac{2x}{x+3}\cdot\frac{2x}{2x} + \frac{9}{2x}\cdot\frac{(x+3)}{(x+3)}$$

$$= \frac{4x^2}{2x(x+3)} + \frac{9x+27}{2x(x+3)}$$

$$= \frac{4x^2+9x+27}{2x(x+3)},\ x \neq -4, -3, 0$$

b) $\frac{2x^2 + 16x + 14}{3x^2 + 30x + 27} \cdot \frac{x^2 + 7x - 18}{x^2 + 4x - 21} - \frac{x^2}{x^2 + x - 12}$

$= \frac{2(x^2 + 8x + 7)}{3(x^2 + 10x + 9)} \cdot \frac{(x + 9)(x - 2)}{(x + 7)(x - 3)} - \frac{x^2}{(x + 4)(x - 3)}$

$= \frac{2\cancel{(x + 7)}\cancel{(x + 1)}}{3\cancel{(x + 9)}\cancel{(x + 1)}} \cdot \frac{\cancel{(x + 9)}(x - 2)}{\cancel{(x + 7)}(x - 3)} - \frac{x^2}{(x + 4)(x - 3)}$

$= \frac{2(x - 2)}{3(x - 3)} - \frac{x^2}{(x + 4)(x - 3)}$

Common denominator:

$3(x - 3)(x + 4)$

$= \frac{2(x - 2)}{3(x - 3)} \cdot \frac{(x + 4)}{(x + 4)} - \frac{x^2}{(x + 4)(x - 3)} \cdot \frac{3}{3}$

$= \frac{(2x - 4)(x + 4)}{3(x - 3)(x + 4)} - \frac{3x^2}{3(x - 3)(x + 4)}$

$= \frac{2x^2 + 4x - 16 - 3x^2}{3(x - 3)(x + 4)}$

$= \frac{-x^2 + 4x - 16}{3(x - 3)(x + 4)}, x \neq -9, -7, -4, -1, 3$

C

14. The expression $\frac{3x - 6}{x(x - 3)}$ can be written as the sum of two rational expressions $\frac{A}{x} + \frac{B}{x - 3}$, where A and B are real numbers. Determine the values of A and B.

$\frac{A}{x} + \frac{B}{x - 3} = \frac{A}{x} \cdot \frac{(x - 3)}{(x - 3)} + \frac{B}{x - 3} \cdot \frac{x}{x}$

$= \frac{Ax - 3A + Bx}{x(x - 3)}$

$= \frac{x(A + B) - 3A}{x(x - 3)}$

Let $\frac{x(A + B) - 3A}{x(x - 3)} = \frac{3x - 6}{x(x - 3)}$

Then, $A + B = 3$ ① and $-3A = -6$ ②

$A = 2$

Substitute $A = 2$ in equation ①.

$2 + B = 3$

$B = 1$

So, the values are $A = 2$ and $B = 1$.

15. Write $\frac{z^2 - 4z + 5}{z^2 - 1}$ as the sum or difference of two rational expressions with different denominators.

Sample response:

$$\frac{z^2 - 4z + 5}{z^2 - 1} = \frac{z^2 + z - 5z + 5}{(z - 1)(z + 1)}$$

$$= \frac{z\cancel{(z + 1)}}{(z - 1)\cancel{(z + 1)}} - \frac{5\cancel{(z - 1)}}{\cancel{(z - 1)}(z + 1)}$$

$$= \frac{z}{(z - 1)} - \frac{5}{(z + 1)}, z \neq -1, 1$$