Lesson 7.5 Exercises, pages 583–590

A

3. Identify the non-permissible values of the variable in each equation.

a) $5 = \frac{3x}{x}$

$x = 0$

b) $\frac{2}{x - 1} = \frac{6}{x}$

$x = 0$ and $x = 1$

c) $\frac{3}{6 - x} = \frac{2}{x + 5}$

$x = -5$ and $x = 6$

d) $\frac{-1}{3x + 6} = \frac{2x}{x - 2}$

$x = -2$ and $x = 2$

4. Solve each equation.

a) $\frac{4}{5} = \frac{8}{d}$

Non-permissible value: $d = 0$
Common denominator: $5d$

$$5d\left(\frac{4}{5}\right) = 5d\left(\frac{8}{d}\right)$$

$$4d = 40$$
$$d = 10$$

b) $\frac{e}{3} = \frac{12}{e}$

Non-permissible value: $e = 0$
Common denominator: $3e$

$$3e\left(\frac{e}{3}\right) = 3e\left(\frac{12}{e}\right)$$

$$e^2 = 36$$
$$e = 6 \text{ or } e = -6$$

Solutions are $e = -6$ and $e = 6$.

5. Solve each equation.

a) $\frac{3}{2z} = \frac{4}{3z} - \frac{1}{2}$

Non-permissible value: $z = 0$
Common denominator: $6z$

$^{3}\cancel{6z}\left(\frac{3}{\cancel{2z}}\right) = {}^{2}\cancel{6z}\left(\frac{4}{\cancel{3z}}\right) - {}^{3}\cancel{6}z\left(\frac{1}{\cancel{2}}\right)$

$9 = 8 - 3z$

$3z = -1$

$z = -\frac{1}{3}$

b) $2 - \frac{4}{x} = \frac{6}{x^2}$

Non-permissible value: $x = 0$
Common denominator: x^2

$x^2(2) - x^{\cancel{2}}\left(\frac{4}{\cancel{x}}\right) = \cancel{x^2}\left(\frac{6}{\cancel{x^2}}\right)$

$2x^2 - 4x = 6$

$2x^2 - 4x - 6 = 0$

$2(x^2 - 2x - 3) = 0$

$2(x - 3)(x + 1) = 0$

$x = 3$ or $x = -1$

Solutions are $x = -1$ and $x = 3$.

B

6. Solve each equation.

a) $\frac{3}{q - 2} = \frac{5}{q + 4}$

Non-permissible values: $q = 2$ and $q = -4$
Common denominator: $(q - 2)(q + 4)$

$\cancel{(q - 2)}(q + 4)\left(\frac{3}{\cancel{q - 2}}\right) = (q - 2)\cancel{(q + 4)}\left(\frac{5}{\cancel{q + 4}}\right)$

$3q + 12 = 5q - 10$

$22 = 2q$

$q = 11$

b) $\frac{3}{2s - 4} = \frac{3}{s - 2}$

Non-permissible value: $s = 2$
The numerators are the same, so equate the denominators.

$2s - 4 = s - 2$

$s = 2$

$s = 2$ is a non-permissible value, so the equation has no solution.

7. Solve each equation.

a) $\frac{a - 1}{a - 3} = \frac{a + 1}{a - 4}$

Non-permissible values: $a = 3$ and $a = 4$
Common denominator: $(a - 3)(a - 4)$

$\cancel{(a - 3)}(a - 4)\left(\frac{a - 1}{\cancel{a - 3}}\right) = (a - 3)\cancel{(a - 4)}\left(\frac{a + 1}{\cancel{a - 4}}\right)$

$(a - 4)(a - 1) = (a - 3)(a + 1)$

$a^2 - 5a + 4 = a^2 - 2a - 3$

$-3a = -7$

$a = \frac{7}{3}$

b) $\frac{2w + 1}{w - 4} = \frac{4w - 3}{2w + 1}$

Non-permissible values: $w = 4$ and $w = -\frac{1}{2}$

Common denominator: $(w - 4)(2w + 1)$

$$\cancel{(w - 4)}(2w + 1)\left(\frac{2w + 1}{\cancel{w - 4}}\right) = (w - 4)\cancel{(2w + 1)}\left(\frac{4w - 3}{\cancel{2w + 1}}\right)$$

$$(2w + 1)(2w + 1) = (w - 4)(4w - 3)$$

$$4w^2 + 4w + 1 = 4w^2 - 19w + 12$$

$$23w = 11$$

$$w = \frac{11}{23}$$

c) $\frac{6}{2x^2 + 2x} = \frac{x - 2}{x + 1}$

$\frac{6}{2x(x + 1)} = \frac{x - 2}{x + 1}$

Non-permissible values: $x = -1$ and $x = 0$

Common denominator: $2x(x + 1)$

$$\cancel{2x(x + 1)}\left(\frac{6}{\cancel{2x(x + 1)}}\right) = 2x\cancel{(x + 1)}\left(\frac{x - 2}{\cancel{x + 1}}\right)$$

$$6 = 2x^2 - 4x$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x - 3)(x + 1) = 0$$

$$x = 3 \text{ or } x = -1$$

$x = -1$ is a non-permissible value.

So, the only solution is $x = 3$.

8. Here is a student's solution for solving a rational equation. Identify the error in the solution. Write a correct solution.

$$\frac{1}{8} = 1 + \frac{2}{x}$$

$$\frac{1}{8} = \frac{3}{x}$$

$$\cancel{8}x\left(\frac{1}{\cancel{8}}\right) = 8\cancel{x}\left(\frac{3}{\cancel{x}}\right)$$

$$x = 24$$

There is an error in line 2: the student added $1 + \frac{2}{x}$ without using a common denominator. Correct solution:

Non-permissible value: $x = 0$

Common denominator: $8x$

$$\frac{1}{8} = 1 + \frac{2}{x}$$

$$\frac{1}{8} = \frac{x + 2}{x}$$

$$\cancel{8}x\left(\frac{1}{\cancel{8}}\right) = 8\cancel{x}\left(\frac{x + 2}{\cancel{x}}\right)$$

$$x = 8x + 16$$

$$7x = -16$$

$$x = -\frac{16}{7}$$

9. Solve each equation.

a) $\frac{1}{x} + \frac{1}{x-3} = \frac{x-2}{x-3}$

Non-permissible values: $x = 0$ and $x = 3$

$$\frac{1}{x} = \frac{x-2}{x-3} - \frac{1}{x-3}$$
$$\frac{1}{x} = \frac{x-3}{x-3}$$
$$\frac{1}{x} = 1$$
$$x = 1$$

b) $\frac{1}{u-2} + \frac{u-1}{u^2-4} = \frac{u+4}{u+2}$

$$\frac{1}{u-2} + \frac{u-1}{(u-2)(u+2)} = \frac{u+4}{u+2}$$

Non-permissible values: $u = -2$ and $u = 2$

Common denominator: $(u-2)(u+2)$

$$\cancel{(u-2)}(u+2)\left(\frac{1}{\cancel{u-2}}\right) + \cancel{(u-2)(u+2)}\left(\frac{u-1}{\cancel{(u-2)(u+2)}}\right) = (u-2)\cancel{(u+2)}\left(\frac{u+4}{\cancel{u+2}}\right)$$
$$u + 2 + u - 1 = (u-2)(u+4)$$
$$2u + 1 = u^2 + 2u - 8$$
$$u^2 - 9 = 0$$
$$u = 3 \text{ or } u = -3$$

Solutions are $u = -3$ and $u = 3$.

c) $\frac{6b^2}{b^2-25} + \frac{4b}{5-b} = \frac{b}{b+5}$

$$\frac{6b^2}{(b-5)(b+5)} - \frac{4b}{b-5} = \frac{b}{b+5}$$

Non-permissible values: $b = -5$ and $b = 5$

Common denominator: $(b-5)(b+5)$

$$\cancel{(b-5)(b+5)}\left(\frac{6b^2}{\cancel{(b-5)(b+5)}}\right) - \cancel{(b-5)}(b+5)\left(\frac{4b}{\cancel{b-5}}\right) = (b-5)\cancel{(b+5)}\left(\frac{b}{\cancel{b+5}}\right)$$
$$6b^2 - 4b(b+5) = b^2 - 5b$$
$$6b^2 - 4b^2 - 20b = b^2 - 5b$$
$$b^2 - 15b = 0$$
$$b(b-15) = 0$$
$$b = 0 \text{ or } b = 15$$

Solutions are $b = 0$ and $b = 15$.

d) $\frac{3z - 1}{2z + 1} + \frac{1}{6} = \frac{2z - 1}{2z + 1} + \frac{z + 1}{z + 3}$

$$\frac{3z - 1}{2z + 1} - \frac{2z - 1}{2z + 1} + \frac{1}{6} = \frac{z + 1}{z + 3}$$
$$\frac{z}{2z + 1} + \frac{1}{6} = \frac{z + 1}{z + 3}$$

Non-permissible values: $z = -\frac{1}{2}$ and $z = -3$

Common denominator: $6(2z + 1)(z + 3)$

$$6\cancel{(2z + 1)}(z + 3)\left(\frac{z}{\cancel{2z + 1}}\right) + \cancel{6}(2z + 1)(z + 3)\left(\frac{1}{\cancel{6}}\right) = 6(2z + 1)\cancel{(z + 3)}\left(\frac{z + 1}{\cancel{z + 3}}\right)$$

$$6z(z + 3) + (2z + 1)(z + 3) = (12z + 6)(z + 1)$$
$$6z^2 + 18z + 2z^2 + 7z + 3 = 12z^2 + 18z + 6$$
$$4z^2 - 7z + 3 = 0$$
$$(4z - 3)(z - 1) = 0$$
$$z = \frac{3}{4} \text{ or } z = 1$$

Solutions are $z = \frac{3}{4}$ and $z = 1$.

10. Solve each equation.

a) $\frac{b}{b^2 - 4} = \frac{2}{b^2 - b - 6}$

$$\frac{b}{(b - 2)(b + 2)} = \frac{2}{(b - 3)(b + 2)}$$

Non-permissible values: $b = 2$, $b = -2$, and $b = 3$

Common denominator: $(b - 2)(b + 2)(b - 3)$

$$\cancel{(b - 2)}\cancel{(b + 2)}(b - 3)\left(\frac{b}{\cancel{(b - 2)}\cancel{(b + 2)}}\right) = (b - 2)\cancel{(b + 2)}\cancel{(b - 3)}\left(\frac{2}{\cancel{(b - 3)}\cancel{(b + 2)}}\right)$$

$$b^2 - 3b = 2b - 4$$
$$b^2 - 5b + 4 = 0$$
$$(b - 4)(b - 1) = 0$$
$$b = 4 \text{ or } b = 1$$

Solutions are $b = 4$ and $b = 1$.

b) $\frac{16}{2g^2 + 2g - 12} = \frac{6}{g^2 - 9}$

$$\frac{16}{2(g^2 + g - 6)} = \frac{6}{(g - 3)(g + 3)}$$
$$\frac{16}{2(g + 3)(g - 2)} = \frac{6}{(g - 3)(g + 3)}$$

Non-permissible values: $g = -3$, $g = 2$, and $g = 3$

Common denominator: $2(g + 3)(g - 3)(g - 2)$

$$\cancel{2}\cancel{(g + 3)}(g - 3)\cancel{(g - 2)}\left(\frac{16}{\cancel{2}\cancel{(g + 3)}\cancel{(g - 2)}}\right) = 2\cancel{(g + 3)}\cancel{(g - 3)}(g - 2)\left(\frac{6}{\cancel{(g - 3)}\cancel{(g + 3)}}\right)$$

$$16g - 48 = 12g - 24$$
$$4g = 24$$
$$g = 6$$

c) $\frac{n}{n+1} + \frac{3n+5}{n^2+4n+3} = \frac{2}{n+3}$

$\frac{n}{n+1} + \frac{3n+5}{(n+1)(n+3)} = \frac{2}{n+3}$

Non-permissible values: $n = -1$ and $n = -3$

Common denominator: $(n + 1)(n + 3)$

$$\cancel{(n+1)}(n+3)\left(\frac{n}{\cancel{n+1}}\right) + \cancel{(n+1)(n+3)}\left(\frac{3n+5}{\cancel{(n+1)(n+3)}}\right) = (n+1)\cancel{(n+3)}\left(\frac{2}{\cancel{n+3}}\right)$$

$n^2 + 3n + 3n + 5 = 2n + 2$

$n^2 + 4n + 3 = 0$

$(n + 1)(n + 3) = 0$

$n = -1$ or $n = -3$

$n = -1$ and $n = -3$ are non-permissible values.

So, the equation has no solution.

11. The solutions of the equation $4 = x + \frac{8}{x+m}$ are $x = 3$ and $x = -4$. Determine the value of m. Show how you can verify your answer.

Common denominator: $x + m$

$$(x+m)4 = (x+m)x + \cancel{(x+m)}\left(\frac{8}{\cancel{x+m}}\right)$$

$4x + 4m = x^2 + mx + 8$

$x^2 + mx - 4x + 8 - 4m = 0$

$x^2 + x(m - 4) + (8 - 4m) = 0$

An equation with roots $x = 3$ and $x = -4$ is:

$(x - 3)(x + 4) = 0$

$x^2 + x - 12 = 0$

Compare the equations: $x^2 + x(m - 4) + (8 - 4m) = 0$

$x^2 + x - 12 = 0$

So, $m - 4 = 1$ and $8 - 4m = -12$

$m = 5$ $\quad$ $m = 5$

The value of m is 5.

To verify the answer, substitute $m = 5$ in the original equation, then solve the equation.

12. The measure, d degrees, of each angle in a regular polygon with n sides is given by the equation $d = 180 - \frac{360}{n}$.

a) What is the measure of each angle in a regular polygon with 15 sides?

Substitute $n = 15$:

$d = 180 - \frac{360}{15}$

$d = 180 - 24$

$d = 156$

Each angle measures 156°.

b) When each angle in a regular polygon is 162°, how many sides does the polygon have?

Substitute $d = 162$:

$$162 = 180 - \frac{360}{n}$$
$$18 = \frac{360}{n}$$
$$n(18) = n\left(\frac{360}{n}\right)$$
$$18n = 360$$
$$n = 20$$

The polygon has 20 sides.

13. Without solving the equation, how do you know that the equation $\frac{12}{4x - 4} = \frac{4}{x - 1}$ has no solution?

Simplify the expression on the left.

$$\frac{\overset{3}{\cancel{12}}}{\cancel{4}(x - 1)} = \frac{4}{x - 1}$$
$$\frac{3}{x - 1} = \frac{4}{x - 1}$$

Since the expressions have the same denominator but different numerators, I know the equation has no solution.

C

14. a) Write a rational equation that has 4 as a solution.

Work backward.

$x = 4$	Write 4 as a difference of 2 numbers.
$x = 7 - 3$	Take 3 to the left side.
$x + 3 = 7$	Divide each side by $x + 3$.
$\frac{x + 3}{x + 3} = \frac{7}{x + 3}$	Simplify.
$1 = \frac{7}{x + 3}, x \neq -3$	

b) Write a rational equation that has −3 as a solution and has 3 as an extraneous root.

$\frac{1}{x - 3}$ has $x = 3$ as a non-permissible value.

Let both rational expressions in the equation have denominator $x - 3$.

$$\frac{\quad}{x - 3} = \frac{\quad}{x - 3}$$

Write an equation that has $x = 3$ and $x = -3$ as roots:

$x^2 = 9$

So, the rational equation becomes:

$$\frac{x^2}{x - 3} = \frac{9}{x - 3}$$

This equation has $x = 3$ and $x = -3$ as roots. Since $x = 3$ is a non-permissible value, $x = 3$ is an extraneous root. The only solution is $x = -3$.

15. Solve each equation.

a) $\frac{x+1}{x-3} = \frac{2x}{x+2}$

Non-permissible values: $x = 3$ and $x = -2$
Common denominator: $(x - 3)(x + 2)$

$$\cancel{(x-3)}(x+2)\left(\frac{x+1}{\cancel{x-3}}\right) = (x-3)\cancel{(x+2)}\left(\frac{2x}{\cancel{x+2}}\right)$$

$$x^2 + 3x + 2 = 2x^2 - 6x$$

$$x^2 - 9x - 2 = 0 \quad \text{Use the quadratic formula.}$$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{9 \pm \sqrt{89}}{2}$$

Solutions are $x = \frac{9 + \sqrt{89}}{2}$ and $x = \frac{9 - \sqrt{89}}{2}$.

b) $\frac{w^2}{w+4} = \frac{5}{3}$

Non-permissible value: $w = -4$
Common denominator: $3(w + 4)$

$$3\cancel{(w+4)}\left(\frac{w^2}{\cancel{w+4}}\right) = \cancel{3}(w+4)\left(\frac{5}{\cancel{3}}\right)$$

$$3w^2 = 5w + 20$$

$$3w^2 - 5w - 20 = 0 \quad \text{Use the quadratic formula.}$$

$$w = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-20)}}{2(3)}$$

$$w = \frac{5 \pm \sqrt{265}}{6}$$

Solutions are $w = \frac{5 + \sqrt{265}}{6}$ and $w = \frac{5 - \sqrt{265}}{6}$.

c) $\frac{1}{v} + \frac{1}{v-4} = 2$

Non-permissible values: $v = 0$ and $v = 4$
Common denominator: $v(v - 4)$

$$\cancel{v}(v-4)\left(\frac{1}{\cancel{v}}\right) + v\cancel{(v-4)}\left(\frac{1}{\cancel{v-4}}\right) = v(v-4)(2)$$

$$v - 4 + v = 2v^2 - 8v$$

$$2v^2 - 10v + 4 = 0$$

$$v^2 - 5v + 2 = 0 \quad \text{Use the quadratic formula.}$$

$$v = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$

$$v = \frac{5 \pm \sqrt{17}}{2}$$

Solutions are $v = \frac{5 + \sqrt{17}}{2}$ and $v = \frac{5 - \sqrt{17}}{2}$.