REVIEW, pages 606–609

7.1

1. For each rational expression below:
- Identify the non-permissible values.
- Write the expressions in simplest form.
- Write an equivalent expression.

a) $\frac{2x + 16}{x^2 - 64} = \frac{2(x + 8)}{(x + 8)(x - 8)}$

Non-permissible values: $x = -8$ and $x = 8$

$\frac{2\cancel{(x + 8)}}{\cancel{(x + 8)}(x - 8)} = \frac{2}{x - 8}, x \neq -8, 8$

$\frac{2(x + 8)}{(x + 8)(x - 8)} = \frac{2(x + 8)}{(x + 8)(x - 8)} \cdot \frac{x + 8}{x + 8}$

$= \frac{2(x + 8)^2}{(x + 8)^2(x - 8)}, x \neq -8, 8$

b) $\frac{x^2 + x - 2}{-x^2 - 2x + 3} = \frac{(x + 2)(x - 1)}{-(x^2 + 2x - 3)}$

$= \frac{(x + 2)(x - 1)}{-(x + 3)(x - 1)}$

Non-permissible values: $x = -3$ and $x = 1$

$\frac{(x + 2)\cancel{(x - 1)}}{-(x + 3)\cancel{(x - 1)}} = \frac{x + 2}{-x - 3}, x \neq -3, 1$

$\frac{(x + 2)(x - 1)}{-(x + 3)(x - 1)} = \frac{(x + 2)(x - 1)}{-(x + 3)(x - 1)} \cdot \frac{x + 2}{x + 2}$

$= \frac{(x + 2)^2(x - 1)}{-(x + 3)(x - 1)(x + 2)}, x \neq -3, -2, 1$

7.2

2. Simplify each expression.

a) $\frac{25ab^2}{4b} \cdot \frac{10b^3}{5a}$

$= \frac{{}^{5}\cancel{25}\,a\cancel{b^2}^{\cancel{2}}}{{}_{2}\cancel{4}\,\cancel{b}} \cdot \frac{\cancel{10}^{\cancel{5}}b^3}{\cancel{5}\,\cancel{a}}$

$= \frac{25b^4}{2}, a \neq 0, b \neq 0$

b) $\frac{6y^2}{y^2 - 9} \cdot \frac{y + 3}{15y}$

$= \frac{6y^2}{(y - 3)(y + 3)} \cdot \frac{y + 3}{15y}$

$= \frac{{}^{2}\cancel{6}\,y^{\cancel{2}}}{(y - 3)\cancel{(y + 3)}} \cdot \frac{\cancel{y + 3}}{{}_{5}\cancel{15}\,y}$

$= \frac{2y}{5(y - 3)}, y \neq -3, 0, 3$

c) $\frac{n-2}{n} \div \frac{3n-6}{n^2+n}$

$= \frac{n-2}{n} \div \frac{3(n-2)}{n(n+1)}$

$= \frac{\cancel{n-2}}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{3\cancel{(n-2)}}$

$= \frac{n+1}{3}, n \neq -1, 0, 2$

d) $\frac{b^2+4b+3}{3-b} \div \frac{b^2+2b+1}{b^2-9}$

$= \frac{(b+1)(b+3)}{3-b} \div \frac{(b+1)(b+1)}{(b-3)(b+3)}$

$= \frac{(b+1)(b+3)}{3-b} \cdot \frac{(b-3)(b+3)}{(b+1)(b+1)}$

$= \frac{\cancel{(b+1)}(b+3)}{-\cancel{(b-3)}} \cdot \frac{\cancel{(b-3)}(b+3)}{\cancel{(b+1)}(b+1)}$

$= -\frac{(b+3)^2}{b+1}, b \neq -3, -1, 3$

7.3

3. Add or subtract.

a) $\frac{6}{x^3} + \frac{5}{2x^4}$

Common denominator: $2x^4$

$= \frac{6}{x^3} \cdot \frac{2x}{2x} + \frac{5}{2x^4}$

$= \frac{12x}{2x^4} + \frac{5}{2x^4}$

$= \frac{12x+5}{2x^4}, x \neq 0$

b) $\frac{-8}{3ef} - \frac{2}{4f} + \frac{5}{2e^2}$

Common denominator: $12e^2f$

$= \frac{-8}{3ef} \cdot \frac{4e}{4e} - \frac{2}{4f} \cdot \frac{3e^2}{3e^2} + \frac{5}{2e^2} \cdot \frac{6f}{6f}$

$= \frac{-32e}{12e^2f} - \frac{6e^2}{12e^2f} + \frac{30f}{12e^2f}$

$= \frac{-32e-6e^2+30f}{12e^2f}$

$= \frac{2(-16e-3e^2+15f)}{12e^2f}$

$= \frac{-16e-3e^2+15f}{6e^2f}, e \neq 0, f \neq 0$

7.4

4. Add or subtract.

a) $\frac{6}{v-2} + \frac{7}{2v+7}$

Common denominator: $(v-2)(2v+7)$

$= \frac{6}{(v-2)} \cdot \frac{(2v+7)}{(2v+7)} + \frac{7}{(2v+7)} \cdot \frac{(v-2)}{(v-2)}$

$= \frac{12v+42}{(v-2)(2v+7)} + \frac{7v-14}{(v-2)(2v+7)}$

$= \frac{19v+28}{(v-2)(2v+7)}, v \neq -\frac{7}{2}, 2$

b) $\dfrac{2z}{3z^2 + 19z - 14} + \dfrac{3}{9z^2 - 12z + 4}$

$= \dfrac{2z}{(3z - 2)(z + 7)} + \dfrac{3}{(3z - 2)(3z - 2)}$

Common denominator: $(3z - 2)^2(z + 7)$

$= \dfrac{2z}{(3z - 2)(z + 7)} \cdot \dfrac{(3z - 2)}{(3z - 2)} + \dfrac{3}{(3z - 2)(3z - 2)} \cdot \dfrac{(z + 7)}{(z + 7)}$

$= \dfrac{6z^2 - 4z}{(3z - 2)^2(z + 7)} + \dfrac{3z + 21}{(3z - 2)^2(z + 7)}$

$= \dfrac{6z^2 - z + 21}{(3z - 2)^2(z + 7)}, z \neq -7, \dfrac{2}{3}$

7.5

5. Solve each equation.

a) $\dfrac{3}{5f} = 1 - \dfrac{7}{2f}$

Non-permissible value: $f = 0$

Common denominator: $10f$

$\dfrac{3}{5f} = 1 - \dfrac{7}{2f}$

$\dfrac{3}{5f} = \dfrac{2f - 7}{2f}$

$^2\cancel{10f}\left(\dfrac{3}{\cancel{5f}}\right) = {}^5\cancel{10f}\left(\dfrac{2f - 7}{\cancel{2f}}\right)$

$6 = 10f - 35$

$41 = 10f$

$f = \dfrac{41}{10}$

b) $\dfrac{4}{5x - 2} = \dfrac{3}{4x - 1}$

Non-permissible values:

$x = \dfrac{2}{5}$ and $x = \dfrac{1}{4}$

Common denominator:

$(5x - 2)(4x - 1)$

$\cancel{(5x - 2)}(4x - 1)\left(\dfrac{4}{\cancel{5x - 2}}\right)$

$= (5x - 2)\cancel{(4x - 1)}\left(\dfrac{3}{\cancel{4x - 1}}\right)$

$16x - 4 = 15x - 6$

$x = -2$

c) $\dfrac{2p}{p - 1} + \dfrac{p - 5}{p^2 - 1} = 1$

$\dfrac{2p}{p - 1} + \dfrac{p - 5}{(p - 1)(p + 1)} = 1$

Non-permissible values: $p = 1$ and $p = -1$

Common denominator: $(p - 1)(p + 1)$

$\cancel{(p - 1)}(p + 1)\left(\dfrac{2p}{\cancel{p - 1}}\right) + \cancel{(p - 1)}\cancel{(p + 1)}\left(\dfrac{p - 5}{\cancel{(p - 1)}\cancel{(p + 1)}}\right) = (p - 1)(p + 1)(1)$

$2p^2 + 2p + p - 5 = p^2 - 1$

$p^2 + 3p - 4 = 0$

$(p + 4)(p - 1) = 0$

$p = -4$ or $p = 1$

$p = 1$ is a non-permissible value.

So, the only solution is $p = -4$.

7.6

6. Two fishing boats have the same average speed in still water. They leave a dock at the same time. One boat heads upstream and the other heads downstream. At a certain point, boat A is 56 km downstream and boat B is 24 km upstream. The average speed of the current is 8 km/h. What is the average speed of the boats in still water?

Let the average speed of the boats in still water be s km/h.
Average speed of current: 8 km/h
Boat A: downstream
Average speed downstream: $(s + 8)$ km/h
Distance travelled: 56 km
Time for boat A: $\frac{56}{s + 8}$ hours
Boat B: upstream
Average speed upstream: $(s - 8)$ km/h
Distance travelled: 24 km
Time for boat B: $\frac{24}{s - 8}$ hours
Boats take same time to travel these distances.
So, an equation is: $\frac{56}{s + 8} = \frac{24}{s - 8}$, $s > 8$
Non-permissible values: $s = -8$ and $s = 8$
Common denominator: $(s + 8)(s - 8)$

$$\frac{56}{s + 8} = \frac{24}{s - 8}$$

$$\cancel{(s + 8)}(s - 8)\left(\frac{56}{\cancel{s + 8}}\right) = (s + 8)\cancel{(s - 8)}\left(\frac{24}{\cancel{s - 8}}\right)$$

$$56s - 448 = 24s + 192$$

$$32s = 640$$

$$s = 20$$

The average speed of the boats in still water is 20 km/h.