

Lesson 8.1 Exercises, pages 621–629

A

4. Complete each table of values.

a)

x	-3	-2	-1	0	1	2	3
$y = -3x + 2$	11	8	5	2	-1	-4	-7
$y = -3x + 2 $	11	8	5	2	1	4	7

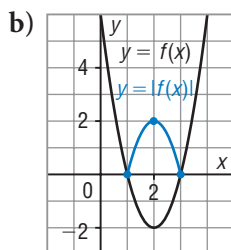
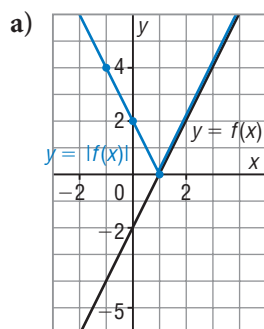
To complete the table for $y = |-3x + 2|$, take the absolute value of each value of $y = -3x + 2$.

b)

x	-4	-3	-2	-1	0	1	2	3	4
$y = 2x^2 - x - 4$	32	17	6	-1	-4	-3	2	11	24
$y = 2x^2 - x - 4 $	32	17	6	1	4	3	2	11	24

To complete the table for $y = |2x^2 - x - 4|$, take the absolute value of each value of $y = 2x^2 - x - 4$.

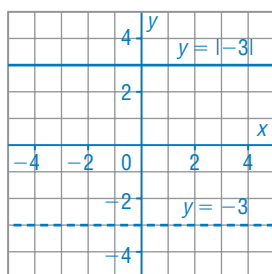
5. The graph of $y = f(x)$ is given. Sketch the graph of $y = |f(x)|$.



The value of y is never negative, so reflect, in the x -axis, the part of each graph that is below the x -axis.

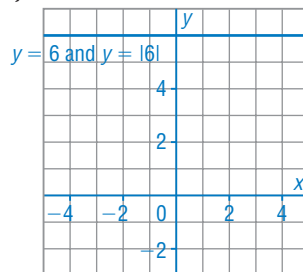
6. Sketch a graph of each absolute value function.

a) $y = |-3|$



Graph the line $y = -3$.
The graph lies below the x -axis, so to graph $y = |-3|$, reflect the entire graph in the x -axis.

b) $y = |6|$



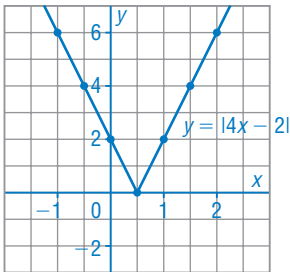
Graph the line $y = 6$.
The graph lies above the x -axis, so the graph of $y = |6|$ is the same as the graph of $y = 6$.

B

7. Complete each table of values, then graph the absolute value function. Identify its intercepts, domain, and range.

a)

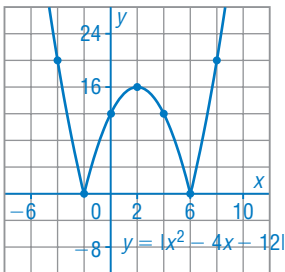
x	-1	-0.5	0	0.5	1	1.5	2
$y = 4x - 2$	-6	-4	-2	0	2	4	6
$y = 4x - 2 $	6	4	2	0	2	4	6



Plot the points, then join them.
 From the graph, the x -intercept is 0.5 and the y -intercept is 2.
 The domain of $y = |4x - 2|$ is $x \in \mathbb{R}$, and the range is $y \geq 0$.

b)

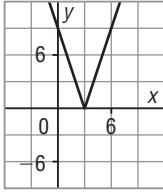
x	-4	-2	0	2	4	6	8
$y = x^2 - 4x - 12$	20	0	-12	-16	-12	0	20
$y = x^2 - 4x - 12 $	20	0	12	16	12	0	20



Plot the points, then join them with a smooth curve.
 From the graph, the x -intercepts are -2 and 6, and the y -intercept is 12.
 The domain of $y = |x^2 - 4x - 12|$ is $x \in \mathbb{R}$, and the range is $y \geq 0$.

8. Write each absolute value function in piecewise notation.

a) $y = |-3x + 9|$



x-intercept: 3

$y = -3x + 9$ when

$-3x + 9 \geq 0$, or $x \leq 3$

$y = -(-3x + 9)$, or

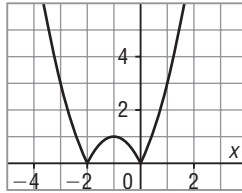
$y = 3x - 9$ when

$-3x + 9 < 0$, or $x > 3$

So, using piecewise notation:

$$y = \begin{cases} -3x + 9, & \text{if } x \leq 3 \\ 3x - 9, & \text{if } x > 3 \end{cases}$$

b) $y = |-(x + 1)^2 + 1|$



x-intercepts: -2 and 0

The graph of $y = -(x + 1)^2 + 1$ opens down.

$-(x + 1)^2 + 1 \geq 0$ for $-2 \leq x \leq 0$.

So, for these domain values,

$|-(x + 1)^2 + 1| = -(x + 1)^2 + 1$.

When $x < -2$ or $x > 0$, the value of

$-(x + 1)^2 + 1$ is negative. So,

for these domain values,

$|-(x + 1)^2 + 1| = -(-(x + 1)^2 + 1)$,

or $(x + 1)^2 - 1$.

So, using piecewise notation:

$$y = \begin{cases} -(x + 1)^2 + 1, & \text{if } -2 \leq x \leq 0 \\ (x + 1)^2 - 1, & \text{if } x < -2 \text{ or } x > 0 \end{cases}$$

9. Write each absolute value function in piecewise notation.

a) $y = |-x + 2|$

$y = -x + 2$ when

$-x + 2 \geq 0$

$-x \geq -2$

$x \leq 2$

$y = -(-x + 2)$,

or $y = x - 2$ when

$-x + 2 < 0$

$-x < -2$

$x > 2$

So, using piecewise notation:

$$y = \begin{cases} -x + 2, & \text{if } x \leq 2 \\ x - 2, & \text{if } x > 2 \end{cases}$$

b) $y = |3x - 5|$

$y = 3x - 5$ when

$3x - 5 \geq 0$

$3x \geq 5$

$x \geq \frac{5}{3}$

$y = -(3x - 5)$,

or $y = -3x + 5$ when

$3x - 5 < 0$

$x < \frac{5}{3}$

So, using piecewise notation:

$$y = \begin{cases} 3x - 5, & \text{if } x \geq \frac{5}{3} \\ -3x + 5, & \text{if } x < \frac{5}{3} \end{cases}$$

c) $y = |x(x - 4)|$

Determine the x -intercepts of the graph of $y = x(x - 4)$.

$$x(x - 4) = 0$$

The x -intercepts are:

$$x = 0 \text{ and } x = 4$$

The graph opens up, so between the x -intercepts, the graph is below the x -axis. For the graph of $y = |x(x - 4)|$:

For $x \leq 0$ or $x \geq 4$, the value of $x(x - 4) \geq 0$

For $0 < x < 4$, the value of $x(x - 4) < 0$;

that is, $y = -x(x - 4)$.

So, using piecewise notation:

$$y = \begin{cases} x(x - 4), & \text{if } x \leq 0 \text{ or } x \geq 4 \\ -x(x - 4), & \text{if } 0 < x < 4 \end{cases}$$

d) $y = |-(x - 1)^2 + 4|$

Determine the x -intercepts of the graph of $y = -(x - 1)^2 + 4$.

$$-(x - 1)^2 + 4 = 0$$

$$-(x - 1)^2 = -4$$

$$(x - 1)^2 = 4$$

The x -intercepts are:

$$x = -1 \text{ and } x = 3$$

The graph opens down, so between the x -intercepts, the graph is above the x -axis. For the graph of

$$y = |-(x - 1)^2 + 4|:$$

For $-1 \leq x \leq 3$, the value of

$$-(x - 1)^2 + 4 \geq 0$$

For $x < -1$ or $x > 3$, the value of

$$-(x - 1)^2 + 4 < 0;$$

that is, $y = -(-(x - 1)^2 + 4)$, or

$$y = (x - 1)^2 - 4,$$

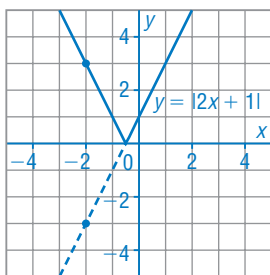
So, using piecewise notation:

$$y = \begin{cases} -(x - 1)^2 + 4, & \text{if } -1 \leq x \leq 3 \\ (x - 1)^2 - 4, & \text{if } x < -1 \text{ or } x > 3 \end{cases}$$

10. Sketch a graph of each absolute value function.

Identify the intercepts, domain, and range.

a) $y = |2x + 1|$



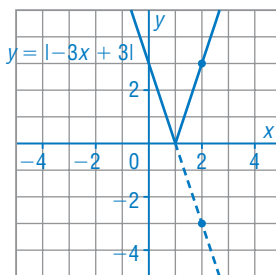
Draw the graph of $y = 2x + 1$.

It has x -intercept -0.5

Reflect, in the x -axis, the part of the graph that is below the x -axis.

From the graph, the x -intercept is -0.5 , the y -intercept is 1 , the domain of $y = |2x + 1|$ is $x \in \mathbb{R}$, and the range is $y \geq 0$.

b) $y = |-3x + 3|$



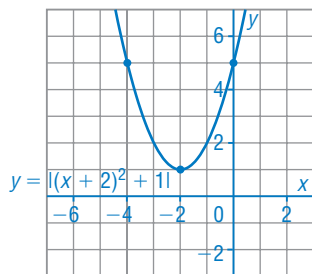
Draw the graph of $y = -3x + 3$.

It has x -intercept 1 .

Reflect, in the x -axis, the part of the graph that is below the x -axis.

From the graph, the x -intercept is 1 , the y -intercept is 3 , the domain of $y = |-3x + 3|$ is $x \in \mathbb{R}$, and the range is $y \geq 0$.

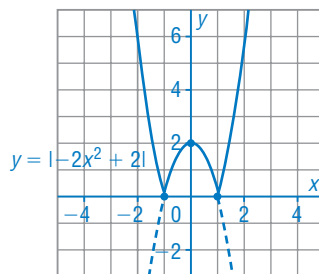
c) $y = |(x + 2)^2 + 1|$



The graph of $y = (x + 2)^2 + 1$ opens up with vertex $(-2, 1)$, so the graph has no x-intercepts. It has y-intercept 5. The graph lies above the x-axis, so the graph of $y = |(x + 2)^2 + 1|$ is the same as the graph of $y = (x + 2)^2 + 1$.

From the graph, there are no x-intercepts, the y-intercept is 5, the domain of $y = |(x + 2)^2 + 1|$ is $x \in \mathbb{R}$ and the range is $y \geq 1$.

d) $y = |-2x^2 + 2|$



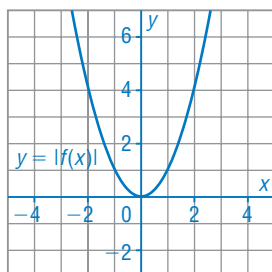
Draw the graph of $y = -2x^2 + 2$. The graph opens down and has x-intercepts 1 and -1 . The vertex is at $(0, 2)$. Reflect, in the x-axis, the part of the graph that is below the x-axis.

From the graph, the x-intercepts are -1 and 1 , the y-intercept is 2. The domain of $y = |-2x^2 + 2|$ is $x \in \mathbb{R}$ and the range is $y \geq 0$.

11. For each description of a quadratic function $y = f(x)$, sketch a graph of $y = |f(x)|$. Describe your thinking.

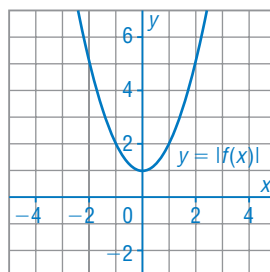
a) the graph of $y = f(x)$ opens up and its vertex is:

i) on the x-axis



The graph of $y = f(x)$ is on or above the x-axis so the graphs of $y = f(x)$ and $y = |f(x)|$ are the same.

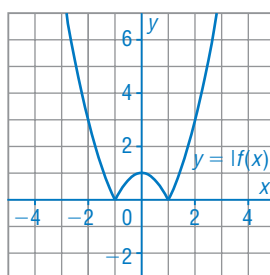
ii) above the x-axis



The graph of $y = f(x)$ is above the x-axis so the graphs of $y = f(x)$ and $y = |f(x)|$ are the same.

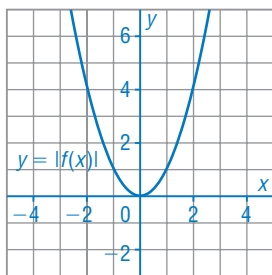
iii) below the x-axis

Part of the graph of $y = f(x)$ is below the x-axis so this part of the graph is reflected in the x-axis to get the graph of $y = |f(x)|$.



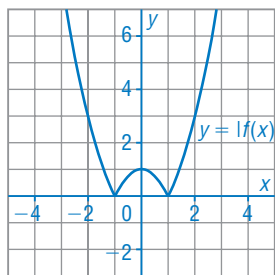
b) the graph of $y = f(x)$ opens down and its vertex is:

i) on the x -axis



The graph of $y = f(x)$ is on or below the x -axis so the graph of $y = f(x)$ is reflected in the x -axis to get the graph of $y = |f(x)|$.

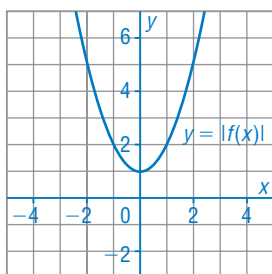
ii) above the x -axis



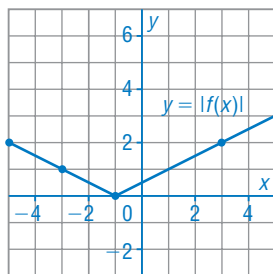
Part of the graph of $y = f(x)$ is below the x -axis so this part of the graph is reflected in the x -axis to get the graph of $y = |f(x)|$.

iii) below the x -axis

The graph of $y = f(x)$ is below the x -axis so the graph is reflected in the x -axis to get the graph of $y = |f(x)|$.

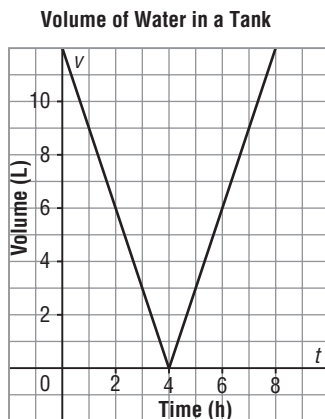


12. The function $y = f(x)$ is linear. Two points on the graph of the absolute value function $y = |f(x)|$ are $(3, 2)$ and $(-1, 0)$. Determine the value of $y = |f(x)|$ when $x = -3$. Explain your strategy.



$(-1, 0)$ is the critical point. Plot the points $(3, 2)$ and $(-1, 0)$ and draw a line through them. The graph of $y = |f(x)|$ is symmetrical about the line $x = -1$. So, reflect the point $(3, 2)$ in the line $x = -1$, then draw a line through $(-1, 0)$ and the reflected point, $(-5, 2)$, to complete the graph of $y = |f(x)|$. From the graph, when $x = -3$, the value of y is 1.

13. This graph represents the volume of water in a 12-L tank over a period of time.



- a) Write the function represented by this graph in piecewise notation. Let v represent the volume in litres and t represent the time in hours.

Equation of line segment for $0 \leq t \leq 4$:

v -intercept: 12 t -intercept: 4

Equation has form: $v = mt + 12$

Use the points (0, 12) and (4, 0) to determine the slope.

$$\begin{aligned} m &= \frac{0 - 12}{4 - 0} \\ &= -3 \end{aligned}$$

So, equation is: $v = -3t + 12$

Equation of line segment for $4 < t \leq 8$:

Passes through: (4, 0) and (8, 12)

Equation has form: $v = mt + b$

$$\begin{aligned} \text{Slope: } m &= \frac{12 - 0}{8 - 4} \\ &= 3 \end{aligned}$$

Use: $v = 3t + b$ Substitute: $t = 4, v = 0$

$$0 = 3(4) + b$$

$$b = -12$$

So, equation is: $v = 3t - 12$

So, using piecewise notation:

$$v = \begin{cases} -3t + 12, & \text{if } 0 \leq t \leq 4 \\ 3t - 12, & \text{if } 4 < t \leq 8 \end{cases}$$

- b) Which absolute value function is represented by this graph?
How do you know?

The equation of the absolute value function is the absolute value of the left segment or the absolute value of the right segment:

$$v = |-3t + 12| \text{ or } v = |3t - 12|$$

- c) Determine the volume of water in the tank after 5.5 h.
What strategy did you use? What other strategy could you use?

Substitute $t = 5.5$ in $v = |3t - 12|$:

$$v = |3(5.5) - 12|$$

$$v = |4.5|$$

$$v = 4.5$$

The volume of water in the tank is 4.5 L.

I could also use the graph to estimate the volume of water after 5.5 h, but this would be an approximate value.

C

14. Two linear functions $y = f(x)$ and $y = h(x)$ have opposite slopes and the same x -intercept.
How will the graphs of $y = |f(x)|$ and $y = |h(x)|$ compare?
Include sketches in your explanation.

Both functions are linear so use the slope-point form of a line.

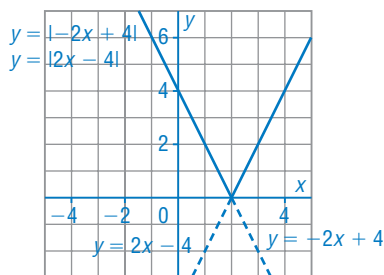
$$y = f(x): y - 0 = m(x - a) \rightarrow y = m(x - a)$$

$$y = g(x): y - 0 = -m(x - a) \rightarrow y = -m(x - a)$$

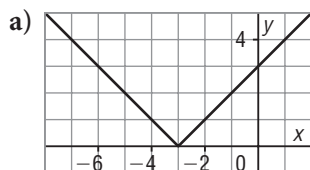
Since $|-m(x - a)| = |m(x - a)|$, the graphs of $y = |f(x)|$ and $y = |h(x)|$ are the same.

For example, the graphs of $y = 2x - 4$ and $y = -2x + 4$ have the same x -intercepts, but opposite slopes.

To graph the absolute value of each function, I reflect the part of the graph below the x -axis in the x -axis. So, the graphs of $y = |2x - 4|$ and $y = |-2x + 4|$ will be the same.



15. Write an equation for each absolute value function. Explain your strategy.



Determine the equation of the line on the right:

y -intercept: 3 x -intercept: -3

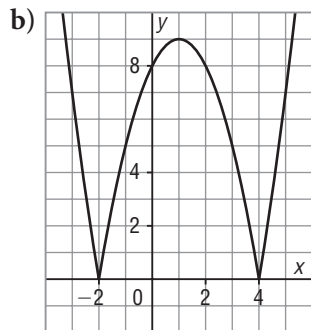
Equation has form: $y = mx + 3$

Use the points $(0, 3)$ and $(-3, 0)$ to determine the slope.

$$m = \frac{0 - 3}{-3 - 0}$$

$$= 1$$

So, an equation is: $y = x + 3$ and an equation for the absolute value function is: $y = |x + 3|$



Either the middle piece of the graph or the two end pieces were reflected in the x -axis. I will assume the middle piece was reflected. So, the graph of the quadratic function opens up and has vertex $(1, -9)$.

So, the equation has the form $y = a(x - 1)^2 - 9$.

Use the point $(4, 0)$ to determine a . Substitute $x = 4$ and $y = 0$.

$$0 = a(4 - 1)^2 - 9$$

$$9 = 9a$$

$$a = 1$$

So, an equation of the quadratic function is $y = (x - 1)^2 - 9$ and an equation of the absolute value function is $y = |(x - 1)^2 - 9|$.