Lesson 8.3 Exercises, pages 657–665

A

3. For each function, write the equation of the corresponding reciprocal function.

a) $y = 5x - 2$

$y = \frac{1}{5x - 2}$

b) $y = 3x$

$y = \frac{1}{3x}$

c) $y = -4$

$y = -\frac{1}{4}$

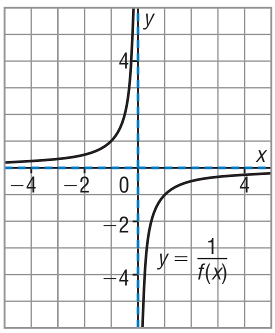
d) $y = \frac{1}{2}$

$y = \frac{1}{\frac{1}{2}}$

$y = 2$

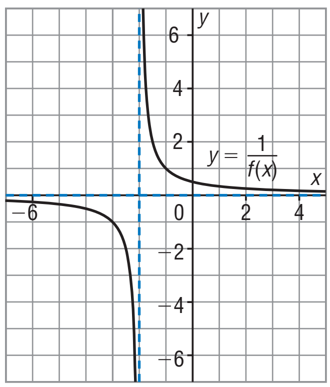
4. Sketch broken lines to represent the vertical and horizontal asymptotes of each graph.

a)



The graph approaches the x-axis, so the line $y = 0$ is a horizontal asymptote. The graph approaches the y-axis, so the line $x = 0$ is a vertical asymptote.

b)



The graph approaches the x-axis, so the line $y = 0$ is a horizontal asymptote. The graph approaches the line $x = -2$, so $x = -2$ is a vertical asymptote.

5. Identify the equation of the vertical asymptote of the graph of each reciprocal function.

a) $y = \frac{1}{2x}$

Let denominator equal 0.

$2x = 0$

$x = 0$

So, graph of $y = \frac{1}{2x}$ has a vertical asymptote at $x = 0$.

b) $y = \frac{1}{-3x + 12}$

Let denominator equal 0.

$-3x + 12 = 0$

$-3x = -12$

$x = 4$

So, graph of $y = \frac{1}{-3x + 12}$ has a vertical asymptote at $x = 4$.

c) $y = \frac{1}{5x + 15}$

Let denominator equal 0.
$5x + 15 = 0$
$5x = -15$
$x = -3$

So, graph of $y = \frac{1}{5x + 15}$ has a vertical asymptote at $x = -3$.

d) $y = \frac{1}{6x - 3}$

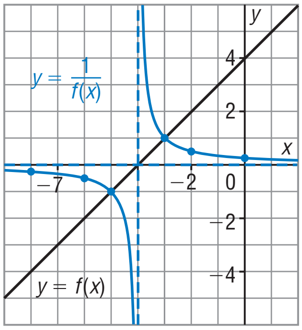
Let denominator equal 0.
$6x - 3 = 0$
$6x = 3$
$x = \frac{1}{2}$

So, graph of $y = \frac{1}{6x - 3}$ has a vertical asymptote at $x = \frac{1}{2}$.

B

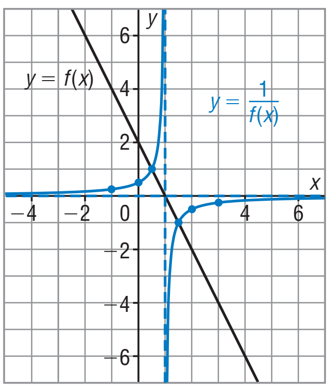
6. Use each graph of $y = f(x)$ to sketch a graph of $y = \frac{1}{f(x)}$. Identify the asymptotes of the graph of the reciprocal function.

a)



Horizontal asymptote: $y = 0$
x-intercept is -4, so vertical asymptote is $x = -4$.
Points $(-3, 1)$ and $(-5, -1)$ are common to both graphs.
Some points on $y = f(x)$ are: $(-2, 2)$, $(0, 4)$, $(-8, -4)$, and $(-6, -2)$. So, points on $y = \frac{1}{f(x)}$ are $(-2, 0.5)$, $(0, 0.25)$, $(-8, -0.25)$, and $(-6, -0.5)$.

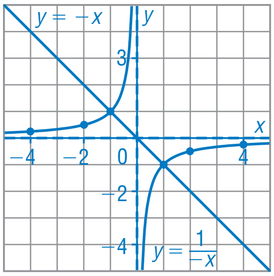
b)



Horizontal asymptote: $y = 0$
x-intercept is 1, so vertical asymptote is $x = 1$.
Points $(0.5, 1)$ and $(1.5, -1)$ are common to both graphs.
Some points on $y = f(x)$ are: $(2, -2)$, $(3, -4)$, $(0, 2)$, and $(-1, 4)$.
So, points on $y = \frac{1}{f(x)}$ are $(2, -0.5)$, $(3, -0.25)$, $(0, 0.5)$, and $(-1, 0.25)$.

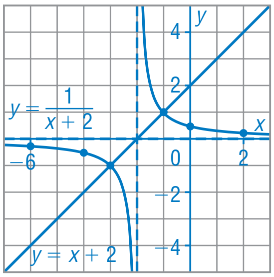
7. For each pair of functions, use a graph of the linear function to sketch a graph of the reciprocal function.
State the domain and range of each reciprocal function.

a) $y = -x$ and $y = \frac{1}{-x}$



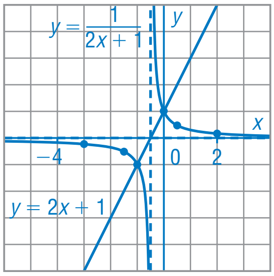
The graph of $y = -x$ has slope -1 and y-intercept 0.
The graph of $y = \frac{1}{-x}$ has horizontal asymptote $y = 0$ and vertical asymptote $x = 0$.
Points $(-1, 1)$ and $(1, -1)$ are common to both graphs.
Some points on $y = -x$ are $(-2, 2)$, $(-4, 4)$, $(2, -2)$, and $(4, -4)$.
So, points on $y = \frac{1}{-x}$ are $(-2, 0.5)$, $(-4, 0.25)$, $(2, -0.5)$, and $(4, -0.25)$.
From the graph, $y = \frac{1}{-x}$ has domain $x \in \mathbb{R}, x \neq 0$ and range: $y \in \mathbb{R}, y \neq 0$.

b) $y = x + 2$ and $y = \frac{1}{x + 2}$



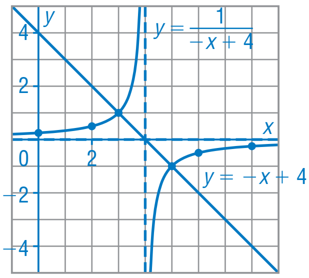
The graph of $y = x + 2$ has slope 1 and y-intercept 2.
The graph of $y = \frac{1}{x + 2}$ has horizontal asymptote $y = 0$ and vertical asymptote $x = -2$.
Points $(-1, 1)$ and $(-3, -1)$ are common to both graphs.
Some points on $y = x + 2$ are $(0, 2)$, $(2, 4)$, $(-4, -2)$, and $(-6, -4)$.
So, points on $y = \frac{1}{x + 2}$ are $(0, 0.5)$, $(2, 0.25)$, $(-4, -0.5)$, and $(-6, -0.25)$.
From the graph, $y = \frac{1}{x + 2}$ has domain $x \in \mathbb{R}, x \neq -2$ and range: $y \in \mathbb{R}, y \neq 0$.

c) $y = 2x + 1$ and $y = \dfrac{1}{2x + 1}$



The graph of $y = 2x + 1$ has slope 2 and y-intercept 1.
The graph of $y = \dfrac{1}{2x + 1}$ has horizontal asymptote $y = 0$ and vertical asymptote $x = -0.5$.
Points (0, 1) and (−1, −1) are common to both graphs.
Some points on $y = 2x + 1$ are (0.5, 2), (2, 5), (−1.5, −2), and (−3, −5).
So, points on $y = \dfrac{1}{2x + 1}$ are (0.5, 0.5), (2, 0.2), (−1.5, −0.5), and (−3, −0.2). From the graph, $y = \dfrac{1}{2x + 1}$ has domain $x \in \mathbb{R}, x \neq -0.5$ and range: $y \in \mathbb{R}, y \neq 0$.

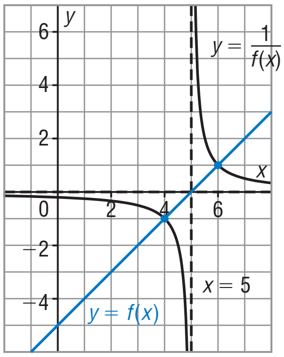
d) $y = -x + 4$ and $y = \dfrac{1}{-x + 4}$



The graph of $y = -x + 4$ has slope −1 and y-intercept 4.
The graph of $y = \dfrac{1}{-x + 4}$ has horizontal asymptote $y = 0$ and vertical asymptote $x = 4$.
Points (3, 1) and (5, −1) are common to both graphs.
Some points on $y = -x + 4$ are (2, 2), (0, 4), (6, −2), and (8, −4).
So, points on $y = \dfrac{1}{-x + 4}$ are (2, 0.5), (0, 0.25), (6, −0.5), and (8, −0.25). From the graph, $y = \dfrac{1}{-x + 4}$ has domain $x \in \mathbb{R}, x \neq 4$ and range: $y \in \mathbb{R}, y \neq 0$.

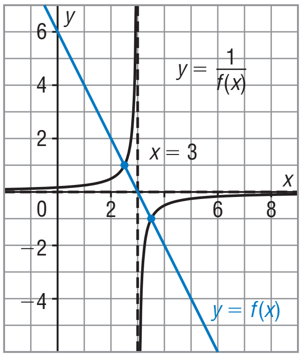
8. Use each graph of $y = \frac{1}{f(x)}$ to graph the linear function $y = f(x)$. Describe the strategy you used.

a)



Vertical asymptote is $x = 5$, so graph of $y = f(x)$ has x-intercept 5.
Mark points at $y = 1$ and $y = -1$ on graph of $y = \frac{1}{f(x)}$, then draw a line through these points for the graph of $y = f(x)$.

b)



Vertical asymptote is $x = 3$, so graph of $y = f(x)$ has x-intercept 3.
Mark points at $y = 1$ and $y = -1$ on graph of $y = \frac{1}{f(x)}$, then draw a line through these points for the graph of $y = f(x)$.

9. Use graphing technology. Graph each pair of functions on the same screen. State the domain and range of each reciprocal function.

a) $y = 3x + 5$ and $y = \frac{1}{3x + 5}$

The graph of $y = 3x + 5$ has x-intercept $-\frac{5}{3}$, so the graph of $y = \frac{1}{3x + 5}$ has vertical asymptote $x = -\frac{5}{3}$. The reciprocal function has domain $x \in \mathbb{R}, x \neq -\frac{5}{3}$ and range: $y \in \mathbb{R}, y \neq 0$.

b) $y = -5x + 0.5$ and $y = \frac{1}{-5x + 0.5}$

Using the Intersect feature, the graph of $y = -5x + 0.5$ has x-intercept 0.1, so the graph of $y = \frac{1}{-5x + 0.5}$ has vertical asymptote $x = 0.1$. The reciprocal function has domain $x \in \mathbb{R}, x \neq 0.1$ and range: $y \in \mathbb{R}, y \neq 0$.

10. **a)** The function $y = f(x)$ is linear. Is it possible for $y = \frac{1}{f(x)}$ to be a horizontal line? If so, how do the domain and range of the reciprocal function compare to the domain and range of $f(x)$?

Yes, if the linear function is a horizontal line of the form $y = c, c \neq 0$, then the graph of the reciprocal function $y = \frac{1}{c}$ is also a horizontal line. The domain of both functions is $x \in \mathbb{R}$. The range of the linear function is $y = c$ and the range of the reciprocal function is $y = \frac{1}{c}$.

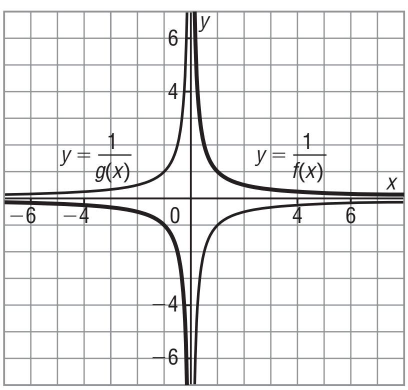
b) The function $y = f(x)$ is linear. Is it possible for $y = \frac{1}{f(x)}$ to be undefined for all real values of x? Explain your thinking.

Yes, if the linear function is the horizontal line $y = 0$, then the graph of the reciprocal function $y = \frac{1}{0}$ is undefined.

11. The reciprocal of a linear function has a vertical asymptote $x = \frac{3}{4}$. What is an equation for the reciprocal function?

The equation of the reciprocal function has the form $y = \frac{1}{mx + b}$.
The vertical asymptote is $x = \frac{3}{4}$, so $mx + b = 0$ when $x = \frac{3}{4}$.
$m\left(\frac{3}{4}\right) + b = 0$
Choose a value for m.
When $m = 4$, $3 + b = 0$, or $b = -3$.
So, an equation for the function is: $y = \frac{1}{4x - 3}$

12. Two linear functions with opposite slopes were used to create graphs of the reciprocal functions $y = \frac{1}{f(x)}$ and $y = \frac{1}{g(x)}$.



a) Which linear function has a positive slope?

The graph of $y = \frac{1}{f(x)}$ goes through the points (1, 1) and (−1, −1). A line through these points goes up to the right, so $y = f(x)$ has a positive slope.

b) Which linear function has a negative slope?
Explain your reasoning.

The graph of $y = \frac{1}{g(x)}$ goes through the points (1, −1) and (−1, 1). A line through these points goes down to the right, so $y = g(x)$ has a negative slope.

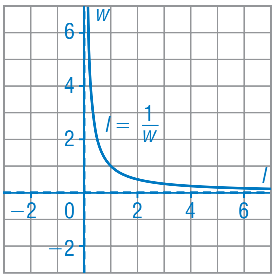
13. a) Write a reciprocal function that describes the length, l metres, of a rectangle with area 1 m^2, as a function of its width, w metres.

Use the formula for the area, A, of a rectangle with length l and width w:
$A = lw$ Substitute: $A = 1$
$1 = lw$
$l = \frac{1}{w}$

b) What are the domain and range of the reciprocal function in part a?

Both length and width are positive. The reciprocal function has vertical asymptote $w = 0$ and horizontal asymptote $l = 0$. So, the domain is $w \in \mathbb{R}$, $w > 0$ and the range is $l \in \mathbb{R}$, $l > 0$.

c) Graph the reciprocal function in part a. Describe the graph. How does it differ from the graphs of other reciprocal functions you have seen? Explain.



Both $l > 0$ and $w > 0$, so the graph of $l = \frac{1}{w}$ is in Quadrant 1. The arm of the graph that would be in Quadrant 3 is not included because both length and width must be positive.

14. A linear function has the form $y = ax + b$, $a \neq 0$. Why does the graph of its reciprocal function always have a vertical and a horizontal asymptote?

A linear function of the form $y = ax + b$, $a \neq 0$, has an x-intercept of $-\frac{b}{a}$ when $y = 0$, so the graph of its reciprocal has a vertical asymptote at $x = -\frac{b}{a}$. The numerator of a reciprocal function is 1. So, the value of the function cannot be 0 for any value of x. When $|x|$ is very large, $\frac{1}{ax + b}$ is close to 0. So, the x-axis is a horizontal asymptote.

C

15. The graphs of two distinct linear functions $y = f(x)$ and $y = g(x)$ are parallel. Do the graphs of their reciprocal functions intersect? How do you know?

The functions are parallel so $y = f(x)$ and $y = g(x)$ have the same slope.
Let the equations of the lines be $y = ax + b$ and $y = ax + c$.
By graphing some examples on my calculator, it seems that the graphs never intersect.
I have to show that there is no value of x for which $\frac{1}{ax + b} = \frac{1}{ax + c}$.
Assume there is a value of x, $x = d$, where $\frac{1}{a(d) + b} = \frac{1}{a(d) + c}$
Then, $ad + c = ad + b$
$c = b$
But, $b \neq c$ because the linear functions are distinct. So, the graphs of the reciprocal functions do not intersect.

16. a) How can you tell without graphing whether the graphs of $y = \frac{1}{x - 2}$ and $y = \frac{1}{-x + 4}$ intersect?

I can equate the two reciprocals to determine if there is a value of x for which the y-values are the same. If there is, then the graphs intersect.

b) Determine the coordinates of any points of intersection.

The y-values are equal when:

$$\frac{1}{x - 2} = \frac{1}{-x + 4}$$
$$x - 2 = -x + 4$$
$$2x = 6$$
$$x = 3$$

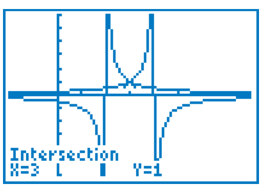
Substitute $x = 3$ in $y = \frac{1}{x - 2}$:

$$y = \frac{1}{3 - 2}$$
$$y = 1$$

The graphs of the reciprocal functions intersect at (3, 1).

c) Use graphing technology. Graph the functions to verify your answer.

The graphs of the reciprocal functions intersect at (3, 1).



17. Determine the equation of the linear function $y = f(x)$ you graphed in question 8, part a.

The linear function has x-intercept 5 and y-intercept -5.
So, the equation has the form $y = mx - 5$.
The line passes through (6, 1) so substitute $x = 6$ and $y = 1$.

$1 = m(6) - 5$

$6 = 6m$

$m = 1$

So, the equation on the linear function is: $y = x - 5$