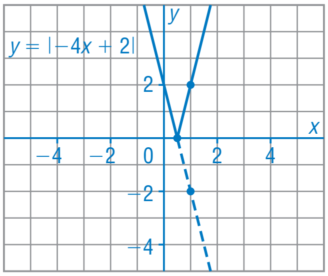
REVIEW, pages 692–697

8.1

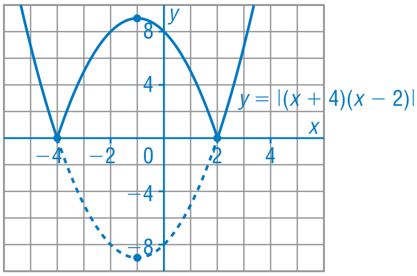
1. Sketch a graph of each absolute function.
Identify the intercepts, domain, and range.

a) $y = |-4x + 2|$



Draw the graph of $y = -4x + 2$.
It has x-intercept 0.5.
Reflect, in the x-axis, the part of the graph that is below the x-axis.
From the graph, the x-intercept is 0.5, the y-intercept is 2, the domain of $y = |-4x + 2|$ is $x \in \mathbb{R}$, and the range is $y \geq 0$.

b) $y = |(x + 4)(x - 2)|$



Draw the graph of $y = (x + 4)(x - 2)$.
It has x-intercepts -4 and 2. The axis of symmetry is $x = -1$, so the vertex is at $(-1, -9)$.
Reflect, in the x-axis, the part of the graph that is below the x-axis.
From the graph, the x-intercepts are -4 and 2, the y-intercept is 8, the domain of $y = |(x + 4)(x - 2)|$ is $x \in \mathbb{R}$, and the range is $y \geq 0$.

2. Write each absolute value function in piecewise notation.

a) $y = |-x - 9|$

$y = -x - 9$ when
$-x - 9 \geq 0$
$-x \geq 9$
$x \leq -9$
$y = -(-x - 9)$,
or $y = x + 9$ when
$-x - 9 < 0$
$-x < 9$
$x > -9$
So, using piecewise notation:

$$y = \begin{cases} -x - 9, & \text{if } x \leq -9 \\ x + 9, & \text{if } x > -9 \end{cases}$$

b) $y = |2x(x + 5)|$

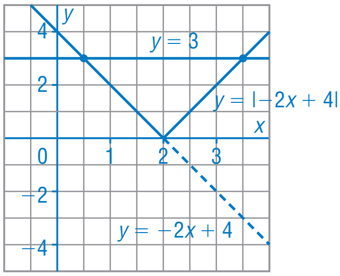
The x-intercepts of the graph of $y = 2x(x + 5)$ are $x = 0$ and $x = -5$.
The graph opens up, so between the x-intercepts, the graph is below the x-axis. For the graph of $y = |2x(x + 5)|$:
For $x \leq -5$ or $x \geq 0$, the value of $2x(x + 5) \geq 0$
For $-5 < x < 0$, the value of $2x(x + 5) < 0$; that is, $y = -2x(x + 5)$.
So, using piecewise notation:

$$y = \begin{cases} 2x(x + 5), & \text{if } x \leq -5 \text{ or } x \geq 0 \\ -2x(x + 5), & \text{if } -5 < x < 0 \end{cases}$$

8.2

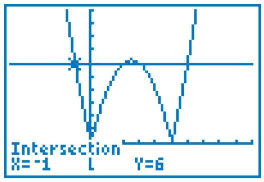
3. Solve by graphing.

a) $3 = |-2x + 4|$



To graph $y = |-2x + 4|$, graph $y = -2x + 4$, then reflect, in the x-axis, the part of the graph that is below the x-axis. The line $y = 3$ intersects $y = |-2x + 4|$ at (0.5, 3) and (3.5, 3). So, the solutions are $x = 0.5$ and $x = 3.5$.

b) $|x^2 - 5x| = 6$



Enter $y = |x^2 - 5x|$ and $y = 6$ in the graphing calculator.
The line $y = 6$ appears to intersect $y = |x^2 - 5x|$ at 4 points: (−1, 6), (2, 6), (3, 6), and (6, 6).
So, the equation has 4 solutions: $x = -1$, $x = 2$, $x = 3$, and $x = 6$

4. Use algebra to solve each equation.

a) $2 = |(x - 1)^2 - 2|$

When $(x - 1)^2 - 2 \geq 0$:
$2 = (x - 1)^2 - 2$
$4 = (x - 1)^2$
$x = 3$ or $x = -1$

When $(x - 1)^2 - 2 < 0$:
$2 = -((x - 1)^2 - 2)$
$-2 = (x - 1)^2 - 2$
$0 = (x - 1)^2$
$x = 1$

So, $x = -1$, $x = 1$, and $x = 3$ are the solutions.

b) $2x = \frac{1}{2}|3x - 5|$

$4x = |3x - 5|$

$4x = 3x - 5$
if $3x - 5 \geq 0$
that is, if $x \geq \frac{5}{3}$

$4x = -(3x - 5)$
if $3x - 5 < 0$
that is, if $x < \frac{5}{3}$

When $x \geq \frac{5}{3}$:
$4x = 3x - 5$
$x = -5$

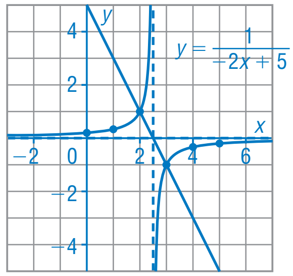
When $x < \frac{5}{3}$:
$4x = -(3x - 5)$
$4x = -3x + 5$
$7x = 5$
$x = \frac{5}{7}$

-5 is not greater than or equal to $\frac{5}{3}$, so -5 is not a solution.
$\frac{5}{7} < \frac{5}{3}$ so $\frac{5}{7}$ is a root.
The solution is $x = \frac{5}{7}$.

8.3

5. Identify the equation of the vertical asymptote of the graph of $y = \frac{1}{-2x + 5}$, then graph the function.

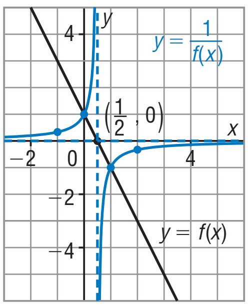
The graph of $y = -2x + 5$ has slope -2, x-intercept $\frac{5}{2}$, and y-intercept 5. The graph of $y = \frac{1}{-2x + 5}$ has a horizontal asymptote $y = 0$ and a vertical asymptote $x = \frac{5}{2}$. Points $(3, -1)$ and $(2, 1)$ are common to both graphs. Some points on $y = -2x + 5$ are $(1, 3)$, $(0, 5)$, $(4, -3)$, and $(5, -5)$. So, points on $y = \frac{1}{-2x + 5}$ are $\left(1, \frac{1}{3}\right)$, $(0, 0.2)$, $\left(4, -\frac{1}{3}\right)$, and $(5, -0.2)$.



6. Use the graph of $y = f(x)$ to sketch a graph of $y = \frac{1}{f(x)}$.

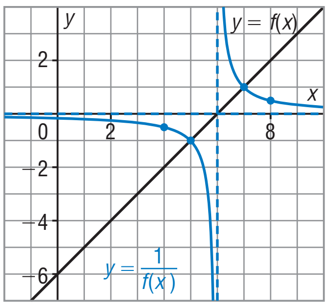
Identify the equations of the asymptotes of the graph of each reciprocal function.

a)



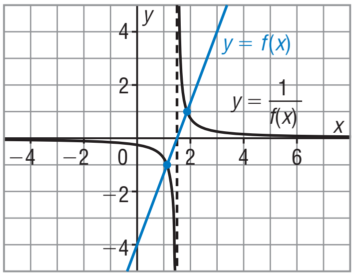
Horizontal asymptote: $y = 0$
x-intercept is $\frac{1}{2}$, so vertical asymptote is $x = \frac{1}{2}$. Points $(0, 1)$ and $(1, -1)$ are common to both graphs. Some points on $y = f(x)$ are: $(-1, 3)$ and $(2, -3)$. So, points on $y = \frac{1}{f(x)}$ are $\left(-1, \frac{1}{3}\right)$ and $\left(2, -\frac{1}{3}\right)$.

b)



Horizontal asymptote: $y = 0$
x-intercept is 6, so vertical asymptote is $x = 6$.
Points $(7, 1)$ and $(5, -1)$ are common to both graphs.
Some points on $y = f(x)$ are: $(8, 2)$ and $(4, -2)$. So, points on $y = \frac{1}{f(x)}$ are $(8, 0.5)$ and $(4, -0.5)$.

7. Use the graph of $y = \frac{1}{f(x)}$ to graph the linear function $y = f(x)$.
Describe your strategy.



Vertical asymptote is $x = 1.5$, so graph of $y = f(x)$ has x-intercept 1.5. Mark points at $y = 1$ and $y = -1$ on graph of $y = \frac{1}{f(x)}$, then draw a line through these points for the graph of $y = f(x)$.

8.4

8. Use a graphing calculator or graphing software.
For which values of q does the graph of
$y = \frac{1}{-(x - 3)^2 + q}$ have:

a) no vertical asymptotes?

Look at the quadratic function $y = -(x - 3)^2 + q$. The graph opens down. For the graph of its reciprocal function to have no vertical asymptotes, the quadratic function must have no x-intercepts. So, the vertex of the quadratic function must be below the x-axis; that is, $q < 0$.

b) one vertical asymptote?

Look at the quadratic function $y = -(x - 3)^2 + q$. The graph opens down. For the graph of its reciprocal function to have one vertical asymptote, the quadratic function must have one x-intercept. So, the vertex of the quadratic function must be on the x-axis; that is, $q = 0$.

c) two vertical asymptotes?

Look at the quadratic function $y = -(x - 3)^2 + q$. The graph opens down. For the graph of its reciprocal function to have two vertical asymptotes, the quadratic function must have two x-intercepts. So, the vertex of the quadratic function must be above the x-axis; that is, $q > 0$.

9. Determine the equations of the vertical asymptotes of the graph of each reciprocal function.

Graph to check the equations.

a) $y = \frac{1}{(x - 2)^2 - 9}$

$(x - 2)^2 - 9 = 0$ when $(x - 2)^2 = 9$; that is, when $x = 5$ or $x = -1$. So, the graph of $y = \frac{1}{(x - 2)^2 - 9}$ has 2 vertical asymptotes, $x = 5$ and $x = -1$. I used my graphing calculator to show that my equations are correct.

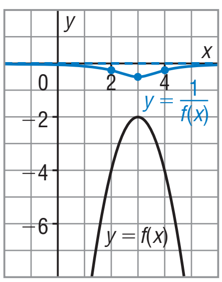
b) $y = \frac{1}{-(x - 2)^2 - 9}$

$-(x - 2)^2 - 9 = 0$ when $(x - 2)^2 = -9$. Since the square of a number is never negative, the graph of $y = -(x - 2)^2 - 9$ has no x-intercepts, and the graph of $y = \frac{1}{-(x - 2)^2 - 9}$ has no vertical asymptotes. I used my graphing calculator to show that my equations are correct.

8.5

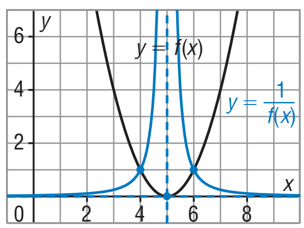
10. On the graph of each quadratic function $y = f(x)$, sketch a graph of the reciprocal function $y = \frac{1}{f(x)}$. Identify the vertical asymptotes, if they exist.

a)



The graph of $y = f(x)$ has no x-intercepts, so the graph of $y = \frac{1}{f(x)}$ has no vertical asymptotes and has Shape 1.
Horizontal asymptote: $y = 0$
Points $(2, -4)$, $(3, -2)$, and $(4, -4)$ lie on $y = f(x)$, so points $(2, -0.25)$, $(3, -0.5)$, and $(4, -0.25)$ lie on $y = \frac{1}{f(x)}$.

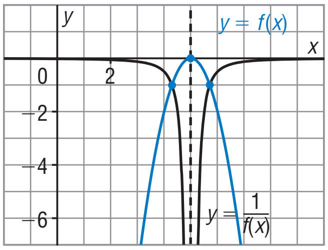
b)



The graph of $y = f(x)$ has 1 x-intercept, so the graph of $y = \frac{1}{f(x)}$ has 1 vertical asymptote, $x = 5$, and has Shape 2.
Horizontal asymptote: $y = 0$
Plot points where the line $y = 1$ intersects the graph of $y = f(x)$. These points are common to both graphs.

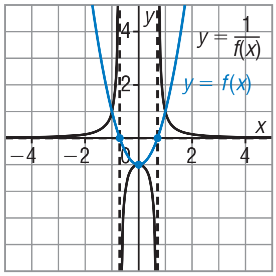
11. On the graph of each reciprocal function $y = \frac{1}{f(x)}$, sketch a graph of the quadratic function $y = f(x)$.

a)



The graph has one vertical asymptote, $x = 5$, so the graph of $y = f(x)$ has vertex (5, 0). The line $y = -1$ intersects the graph at 2 points that are common to both graphs.

b)



The graph has 2 vertical asymptotes, so the graph of $y = f(x)$ has 2 x-intercepts. Plot points where the asymptotes intersect the x-axis. The point $(0, -1)$ is on the line of symmetry, so $(0, -1)$ is the vertex of $y = f(x)$.