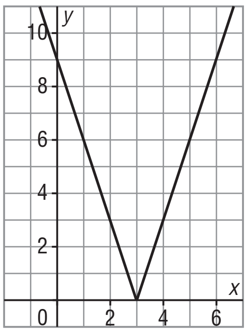
PRACTICE TEST, pages 698–700

1. Multiple Choice Which solution is correct for $|x^2 - 6x + 5| = 5$?

A. no solution
B. $x = 0$
C. $x = 0; x = 6; x = \frac{1}{3}; x = \frac{2}{3}$
D. $x = 0; x = 6$

2. Multiple Choice Which function describes this graph?



A. $y = |3x + 9|$
B. $y = |3x - 9|$
C. $y = |-\frac{1}{3}x + 9|$
D. $y = |9x + 3|$

3. Solve each equation.

a) $|-4x + 4| = 2$

$-4x + 4 = 2$
if $-4x + 4 \geq 0$
that is, if $x \leq 1$
When $x \leq 1$:
$-4x + 4 = 2$
$-4x = -2$
$x = \frac{1}{2}$

$\frac{1}{2} \leq 1$, so this root is a solution.

$-(-4x + 4) = 2$
if $-4x + 4 < 0$
that is, if $x > 1$
When $x > 1$:
$-(-4x + 4) = 2$
$-4x + 4 = -2$
$4x = 6$
$x = \frac{3}{2}$

$\frac{3}{2} > 1$, so this root is a solution.

The solutions are $x = \frac{1}{2}$ and $x = \frac{3}{2}$.

b) $x + 1 = |x^2 - 4x + 5|$

When $x^2 - 4x + 5 \geq 0$:
$x + 1 = x^2 - 4x + 5$
$0 = x^2 - 5x + 4$
$0 = (x - 1)(x - 4)$
$x = 1$ or $x = 4$

When $x^2 - 4x + 5 < 0$:
$x + 1 = -(x^2 - 4x + 5)$
$x + 1 = -x^2 + 4x - 5$
$x^2 - 3x + 6 = 0$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(6)}}{2(1)}$$

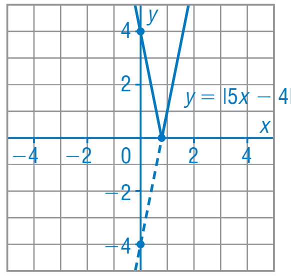
$$x = \frac{3 \pm \sqrt{-15}}{2}$$

This is not a real number.

So, $x = 1$ and $x = 4$ are the solutions.

4. Sketch a graph of each absolute value function. Identify the intercepts, domain, and range. Write the functions in piecewise notation.

a) $y = |5x - 4|$

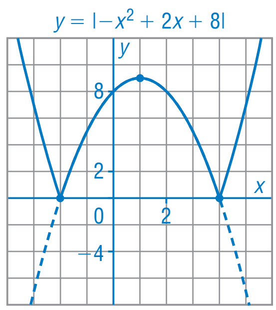


Draw the graph of $y = 5x - 4$.
It has x-intercept $\frac{4}{5}$.
Reflect, in the x-axis, the part of the graph that is below the x-axis.
The x-intercept is $\frac{4}{5}$, the y-intercept is 4, the domain of $y = |5x - 4|$ is $x \in \mathbb{R}$, and the range is $y \geq 0$.

$y = 5x - 4$ when
$5x - 4 \geq 0$, or $x \geq \frac{4}{5}$
$y = -(5x - 4)$, or
$y = -5x + 4$ when
$5x - 4 < 0$, or $x < \frac{4}{5}$
So, using piecewise notation:

$$y = \begin{cases} 5x - 4, & \text{if } x \geq \frac{4}{5} \\ -5x + 4, & \text{if } x < \frac{4}{5} \end{cases}$$

b) $y = |-x^2 + 2x + 8|$



The graph of $y = -x^2 + 2x + 8$, or $y = -(x - 4)(x + 2)$ opens down with x-intercepts 4 and -2.
Its vertex is on the axis of symmetry, $x = 1$, and has coordinates (1, 9).
Reflect, in the x-axis, the part of the graph that is below the x-axis.
From the graph, the x-intercepts are -2 and 4, the y-intercept is 8.
The domain of $y = |-x^2 + 2x + 8|$ is $x \in \mathbb{R}$ and the range is $y \geq 0$.
The graph of the quadratic function opens down, so between the x-intercepts, the graph is above the x-axis.
For the graph of $y = |-x^2 + 2x + 8|$:
For $-2 \leq x \leq 4$, the value of $-x^2 + 2x + 8 \geq 0$
For $x < -2$ or $x > 4$, the value of $-x^2 + 2x + 8 < 0$; that is,
$y = -(-x^2 + 2x + 8)$, or $y = x^2 - 2x - 8$.
So, using piecewise notation:

$$y = \begin{cases} -x^2 + 2x + 8, & \text{if } -2 \leq x \leq 4 \\ x^2 - 2x - 8, & \text{if } x < -2 \text{ or } x > 4 \end{cases}$$

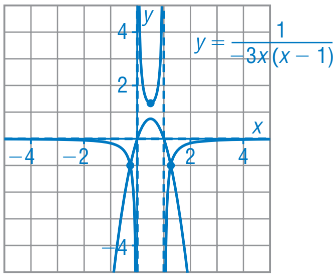
5. Sketch a graph of the function $y = \frac{1}{-3x(x - 1)}$.

Label the asymptotes with their equations.

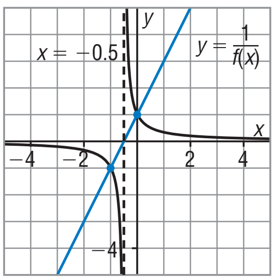
The graph of $y = -3x(x - 1)$ opens down, has x-intercepts 0 and 1, and vertex $\left(0.5, \frac{3}{4}\right)$. The graph of $y = \frac{1}{-3x(x - 1)}$ has vertical asymptotes $x = 0$ and $x = 1$, and a horizontal asymptote $y = 0$. Plot points where the line $y = -1$ intersects the graph of $y = -3x(x - 1)$. These points are common to both graphs.

Point $\left(0.5, -\frac{3}{4}\right)$ lies on $y = -3x(x - 1)$, so point $\left(0.5, \frac{4}{3}\right)$ lies on $y = \frac{1}{-3x(x - 1)}$.

The graph of the reciprocal function has Shape 3.



6. Use the graph of the reciprocal function $y = \frac{1}{f(x)}$ to graph the linear function $y = f(x)$. Describe your strategy.



Vertical asymptote is $x = -0.5$, so graph of $y = f(x)$ has x-intercept -0.5. Mark points at $y = 1$ and $y = -1$ on graph of $y = \frac{1}{f(x)}$, then draw a line through these points for the graph of $y = f(x)$.