Lesson 2.1 Exercises, pages 89–94

A

3. Write the absolute value of each number.

a) 15 $|15| = 15$

b) -7.6 $|-7.6| = 7.6$

c) 0 $|0| = 0$

d) $-\frac{7}{2}$ $\left|-\frac{7}{2}\right| = \frac{7}{2}$

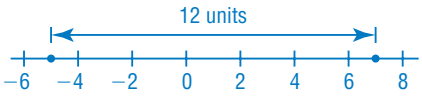
4. Determine the absolute value of the number represented by each point on the number line.

A: $|1| = 1$ B: $|0| = 0$ C: $|-1.5| = 1.5$ D: $|-3| = 3$

5. Use absolute value to determine the distance between each pair of numbers on a number line. Sketch a number line to illustrate the solution.

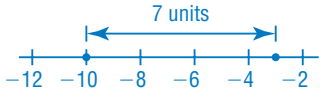
a) -5 and 7

$|7 - (-5)| = |12| = 12$



b) -3 and -10

$|-10 - (-3)| = |-7| = 7$



6. Evaluate.

a) $\sqrt{5^2}$

Since $\sqrt{x^2} = |x|$,
$\sqrt{5^2} = |5|$
$= 5$

b) $\sqrt{(-4)^2}$

Since $\sqrt{x^2} = |x|$,
$\sqrt{(-4)^2} = |-4|$
$= 4$

c) $\sqrt{10.1^2}$

Since $\sqrt{x^2} = |x|$,
$\sqrt{10.1^2} = |10.1|$
$= 10.1$

d) $\sqrt{(-11.4)^2}$

Since $\sqrt{x^2} = |x|$,
$\sqrt{(-11.4)^2} = |-11.4|$
$= 11.4$

B

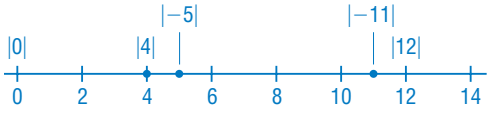
7. a) Determine each absolute value.

$|-11|, |12|, |0|, |4|, |-5|$

$|-11| = 11$ $|12| = 12$ $|0| = 0$
$|4| = 4$ $|-5| = 5$

b) Order the numbers in part a from least to greatest. Show the solution on a number line.

From least to greatest:
$0 < 4 < 5 < 11 < 12$
That is, $|0| < |4| < |-5| < |-11| < |12|$



8. For each pair of numbers, write two expressions to represent the distance between the numbers on a number line, then determine this distance.

a) $-2\frac{3}{4}$ and $5\frac{1}{4}$

$\left|-2\frac{3}{4} - 5\frac{1}{4}\right|$ and $\left|5\frac{1}{4} - \left(-2\frac{3}{4}\right)\right|$, or $\left|5\frac{1}{4} + 2\frac{3}{4}\right|$

$\left|-2\frac{3}{4} - 5\frac{1}{4}\right| = \left|-\frac{11}{4} - \frac{21}{4}\right|$
$= \left|-\frac{32}{4}\right|$
$= |-8|$
$= 8$

The numbers are 8 units apart on a number line.

b) $-5\frac{3}{5}$ and $-3\frac{7}{10}$

$\left|-5\frac{3}{5}-\left(-3\frac{7}{10}\right)\right|$, or $\left|-5\frac{3}{5}+3\frac{7}{10}\right|$,

and $\left|-3\frac{7}{10}-\left(-5\frac{3}{5}\right)\right|$, or $\left|-3\frac{7}{10}+5\frac{3}{5}\right|$

$$\begin{aligned}\left|-5\frac{3}{5}+3\frac{7}{10}\right| &= \left|-\frac{28}{5}+\frac{37}{10}\right| \\ &= \left|-\frac{56}{10}+\frac{37}{10}\right| \\ &= \left|-\frac{19}{10}\right| \\ &= \frac{19}{10}, \text{ or } 1\frac{9}{10}\end{aligned}$$

The numbers are $1\frac{9}{10}$ units apart on a number line.

c) 12.47 and -8.23

$|12.47-(-8.23)|$, or $|12.47+8.23|$, and $|-8.23-12.47|$

$$\begin{aligned}|12.47+8.23| &= |20.7| \\ &= 20.7\end{aligned}$$

The numbers are 20.7 units apart on a number line.

9. Order the absolute values of the numbers in this set from greatest to least. Describe the strategy you used.

$8.5, -8\frac{1}{3}, -10.2, -0.2, 0.1, 9.8$

$|8.5| = 8.5$ $\quad \left|-8\frac{1}{3}\right| = 8\frac{1}{3}$ $\quad |-10.2| = 10.2$

$|-0.2| = 0.2$ $\quad |0.1| = 0.1$ $\quad |9.8| = 9.8$

From greatest to least:

$10.2 > 9.8 > 8.5 > 8\frac{1}{3} > 0.2 > 0.1$

That is, $|-10.2|, |9.8|, |8.5|, \left|-8\frac{1}{3}\right|, |-0.2|, |0.1|$

I determined the absolute value of each number, then ordered the results from greatest to least.

10. The square of a number is 36. What is the absolute value of the number? Show your reasoning.

$$\begin{aligned}36 &= 6^2 \\ \sqrt{36} &= \sqrt{6^2} \\ &= |6| \\ &= 6\end{aligned} \qquad \begin{aligned}36 &= (-6)^2 \\ \sqrt{36} &= \sqrt{(-6)^2} \\ &= |-6| \\ &= 6\end{aligned}$$

So, the absolute value of the number is 6.

11. Evaluate.

a) $\sqrt{\left(3\frac{1}{8}\right)^2}$

$= \left|3\frac{1}{8}\right|$

$= 3\frac{1}{8}$

b) $\sqrt{\left(-2\frac{5}{6}\right)^2}$

$= \left|-2\frac{5}{6}\right|$

$= 2\frac{5}{6}$

c) $\sqrt{(9 - 11)^2}$

$= \sqrt{(-2)^2}$
$= |-2|$
$= 2$

d) $\sqrt{(13.5 - 16.1)^2}$

$= \sqrt{(-2.6)^2}$
$= |-2.6|$
$= 2.6$

12. Evaluate.

a) $|5(4) - 3|$

$= |20 - 3|$
$= |17|$
$= 17$

b) $3|10 - 15|$

$= 3|-5|$
$= 3(5)$
$= 15$

c) $10 - |5 - 8|$

$= 10 - |-3|$
$= 10 - 3$
$= 7$

d) $|6 + (-10)| - |5 - 7|$

$= |-4| - |-2|$
$= 4 - 2$
$= 2$

e) $3|-2| - 4|3|$

$= 3(2) - 4(3)$
$= 6 - 12$
$= -6$

f) $-3|4 - 1| + 2|1 - 4|$

$= -3|3| + 2|-3|$
$= -3(3) + 2(3)$
$= -9 + 6$
$= -3$

13. Evaluate.

a) $\sqrt{(5.1 - 2.3)^2} - |5.1 - 2.3|$

$= \sqrt{2.8^2} - |2.8|$
$= |2.8| - |2.8|$
$= 0$

b) $5.1 - 2.3 - |5.1 - 2.3|$

$= 2.8 - |2.8|$
$= 2.8 - 2.8$
$= 0$

c) $\dfrac{|2 - (-8)|}{|3| - |-2|}$

$= \dfrac{|10|}{3 - 2}$
$= \dfrac{10}{1}$
$= 10$

d) $|5 - 4|(5 + 4) - 2(5 + 4)$

$= |1|(9) - 2(9)$
$= 1(9) - 18$
$= 9 - 18$
$= -9$

e) $\frac{2}{3}\left|-\frac{5}{8}-\frac{1}{4}\right|$

$= \frac{2}{3}\left|-\frac{5}{8}-\frac{2}{8}\right|$

$= \frac{2}{3}\left|-\frac{7}{8}\right|$

$= \frac{\cancel{2}}{3}\left(\frac{7}{\cancel{8}_4}\right)$

$= \frac{7}{12}$

f) $\left|1\frac{3}{4}-2\frac{1}{2}\right|-\left|\frac{3}{4}-\frac{1}{2}\right|$

$= \left|\frac{7}{4}-\frac{5}{2}\right|-\left|\frac{3}{4}-\frac{2}{4}\right|$

$= \left|\frac{7}{4}-\frac{10}{4}\right|-\left|\frac{1}{4}\right|$

$= \left|-\frac{3}{4}\right|-\frac{1}{4}$

$= \frac{3}{4}-\frac{1}{4}$

$= \frac{2}{4}$, or $\frac{1}{2}$

14. Evaluate each expression for the given value of x.

a) $|x^2 + 6x - 5|, x = 2$

$= |2^2 + 6(2) - 5|$
$= |4 + 12 - 5|$
$= |11|$
$= 11$

b) $|-3x^2 + 7x + 1|, x = 3$

$= |-3(3)^2 + 7(3) + 1|$
$= |-27 + 21 + 1|$
$= |-5|$
$= 5$

c) $|x^3 - 5x - 2|, x = -3$

$= |(-3)^3 - 5(-3) - 2|$
$= |-27 + 15 - 2|$
$= |-14|$
$= 14$

d) $|5x^3 + 2x + 7|, x = -1$

$= |5(-1)^3 + 2(-1) + 7|$
$= |-5 - 2 + 7|$
$= |0|$
$= 0$

C

15. Two integers, a and b, are between -5 and 5. The sum of the absolute values of the integers is 4. What might the integers be? How many answers are possible?

Integers between -5 and 5 are: $-4, -3, -2, -1, 0, 1, 2, 3, 4$
$|a| + |b| = 4$
So, I test all pairs of these integers using the equation above.
Possible pairs of integers are: $a = 4, b = 0$; $a = 3, b = 1$; $a = 2, b = 2$; $a = 1, b = 3$; $a = 0, b = 4$; $a = -4, b = 0$; $a = -3, b = -1$; $a = -2, b = -2$; $a = -1, b = -3$; $a = 0, b = -4$; $a = -3, b = 1$; $a = -1, b = 3$; $a = 3, b = -1$; $a = 1, b = -3$; $a = -2, b = 2$; $a = 2, b = -2$
There are 16 possible answers.

16. When 7 is added to an integer, x, the absolute value of the sum is 12. Determine a value for x. How many values of x are possible? Show what you did to solve the problem.

Write, then solve an equation: $|x + 7| = 12$

Since $|12| = 12$ and $|-12| = 12$,

then, $x + 7 = 12$ or $x + 7 = -12$

$x = 5$ $x = -19$

So, two values of x are possible: 5 or -19