Properties of Quadratic Functions

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<u>PRE-CALCULUS 11</u>
<u>QUADRATIC FUNCTIONS</u>
<u>PROPERTIES OF QUADRATIC FUNCTIONS</u>

A. <u>Definitions</u>

1. **quadratic function:** any function that can be written in the form:
$y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$. Where a, b and c are constants and $a \neq 0$.

2. **x-intercept:** the place where the curve crosses the x-axis. These are also referred to as the roots or zeros of the function.

3. **y-intercept:** the place where the curve crosses the y-axis. In the form of the quadratic function the c value represents the y-intercept.

4. **vertex:** the highest or lowest point of a quadratic function.

5. **axis of symmetry:** the imaginary line, through the vertex, that divides the quadratic function into two perfect halves.

6. **maximum/minimum:** the highest or lowest point of a quadratic function.

B. <u>Properties of a Quadratic Function</u>

Remember the important properties of a quadratic function.

The axis of symmetry is always represented by an equation

$$X = ?$$
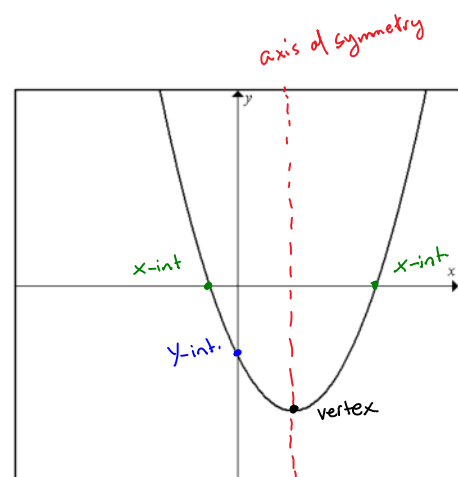
C. Maximum/Minimum Points

Remember that the vertex of a quadratic function will represent the highest/lowest point of the parabola. Whether the graph will have a maximum or a minimum point is determined by the sign of the "a" value in the function.

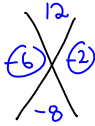**Positive "a" Value**

Upright Parabola
Vertex is Minimum.

Negative "a" Value

Inverted Parabola.
Vertex is Maximum

D. Examples

For each of the following quadratic functions, determine the x-intercepts (zeros), y-intercept, vertex, the equation of the axis of symmetry, and whether the graph is a maximum or minimum.

12

(-6)   (-2)

-8

1) $y = x^2 - 8x + 12$

$0 = x^2 - 8x + 12$

$0 = (x - 6)(x - 2)$

$0 = x - 6 \qquad 0 = x - 2$
$ +6 +6 \qquad +2 +2$

$\boxed{x-int\ (6,0)\ \&\ (2,0)}$

$x-coord = \frac{6+2}{2} = \frac{4}{1}$

$y = x^2 - 8x + 12$
$y = (4)^2 - 8(4) + 12$
$y-coord = -\frac{4}{1}$

$\boxed{Vertex\ (4,-4)}$

$\boxed{y-int\ (0,12)}$

$(4,-4)$

$\boxed{Axis\ of\ Symmetry \quad x = 4.}$

$\boxed{Minimum}$

2) $f(x) = -2x^2 + 2x + 24$

$$0 = -2x^2 + 2x + 24$$
$$0 = -2(x^2 - x - 12)$$
$$0 = -2(x - 4)(x + 3)$$

$$0 = x - 4 \qquad 0 = x + 3$$
$$+4 \quad +4 \qquad -3 \quad -3$$

$\boxed{x\text{-int } (4,0) \ \& \ (-3,0)}$

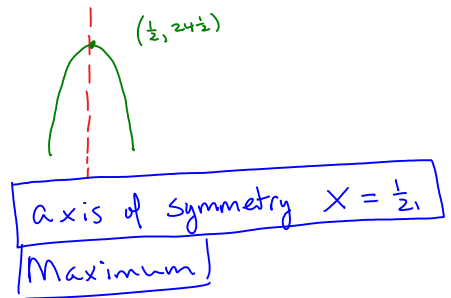$x\text{-coord} = \dfrac{4 + -3}{2} = \dfrac{1}{2}$

$$y = -2x^2 + 2x + 24$$
$$y = -2\left(\tfrac{1}{2}\right)^2 + 2\left(\tfrac{1}{2}\right) + 24$$
$$y\text{-coord} = 24\tfrac{1}{2}$$

$\boxed{\text{vertex } \left(\tfrac{1}{2}, 24\tfrac{1}{2}\right)}$

$\boxed{y\text{-int } (0, 24)}$

$\left(\tfrac{1}{2}, 24\tfrac{1}{2}\right)$

$\boxed{\text{axis of symmetry } \ x = \tfrac{1}{2},}$

$\boxed{\text{Maximum}}$

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3) $y = \dfrac{1}{2}x^2 + 2x - \dfrac{5}{2}$

$$0 = \tfrac{1}{2}x^2 + 2x - \tfrac{5}{2}$$
$$0 = \tfrac{1}{2}(x^2 + 4x - 5)$$
$$0 = \tfrac{1}{2}(x - 1)(x + 5)$$

$$0 = x - 1 \qquad 0 = x + 5$$
$$+1 \qquad\qquad -5$$

$\boxed{x\text{-int } (1,0) \ \& \ (-5,0)}$
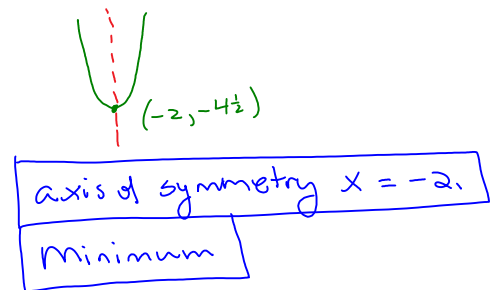
$x\text{-coord} = \dfrac{1 + -5}{2} = -2$

$$y = \tfrac{1}{2}x^2 + 2x - \tfrac{5}{2}$$
$$y = \tfrac{1}{2}(-2)^2 + 2(-2) - \tfrac{5}{2}$$
$$y\text{-coord} = -4\tfrac{1}{2}$$

$\boxed{\text{vertex } \left(-2, -4\tfrac{1}{2}\right)}$

$\boxed{y\text{-int } \left(0, \tfrac{-5}{2}\right)}$

$(-2, -4\tfrac{1}{2})$

$\boxed{\text{axis of symmetry } \ x = -2,}$

$\boxed{\text{Minimum}}$

Assignment : Properties of Quadratic Functions Assignment #1 - 12

A. For each of the following quadratic functions, determine the coordinates of the x-intercepts, y-intercept and vertex, then graph the function and state the domain and range.

1) $y = x^2 + 2x - 3$

2) $y = -2x^2 + 8$

3) $y = x^2 - 6x + 8$

4) $y = -x^2 - 6x - 5$

B. For each of the following quadratic functions, determine, the vertex, the equation for the axis of symmetry whether the graph is a maximum or minimum.

5) $y = x^2 + 10x + 21$

6) $y = x^2 + 6x - 7$

7) $y = -2x^2 - 12x + 32$

8) $y = x^2 + 3x - 4$

9) $y = \frac{1}{2}x^2 + 2x - 6$

10) $y = 3x^2 - 9x - 30$

11) $y = -x^2 - x + 6$

12) $y = \frac{1}{3}x^2 - \frac{5}{3}x + \frac{4}{3}$

Answers

1) x-int $(-3,0)$, $(1,0)$
   y-int $(0,-3)$
   vertex $(-1,-4)$
   Domain $x \in R$
   Range $y \geq -4$

2) x-int $(-2,0)$, $(2,0)$
   y-int $(0,8)$
   vertex $(0,8)$
   Domain $x \in R$
   Range $y \leq 8$

3) x-int $(2,0)$, $(4,0)$
   y-int $(0,8)$
   vertex $(3,-1)$
   Domain $x \in R$
   Range $y \geq -1$

4) x-int $(-1,0)$, $(-5,0)$
   y-int $(0,-5)$
   vertex $(-3,4)$
   Domain $x \in R$
   Range $y \leq 4$

5) vertex $(-5,-4)$
   axis of sym $x = -5$
   Minimum

6) vertex $(-3,-16)$
   axis of sym $x = -3$
   Minimum

7) vertex $(-3,50)$
   axis of sym $x = -3$
   Maximum

8) vertex $(-1.5,-6.25)$
   axis of sym $x = -1.5$
   Minimum

9) vertex $(-2,-8)$
   axis of sym $x = -2$
   Minimum

10) vertex $(1.5,-36.75)$
    axis of sym $x = 1.5$
    Minimum

11) vertex $(-0.5,6.25)$
    axis of sym $x = -0.5$
    Maximum

12) vertex $(2.5,-0.75)$
    axis of sym $x = 2.5$
    Minimum