Rational Exponents

June-05-19
11:23 AM

A. <u>Definitions</u>

  1. **Rational Number:** a number which can be written in the form of a fraction.

  2. **Rational Exponent:** an exponent which is written in the form of a fraction.

  3. **Radical:** a mathematical expression containing a root symbol.

B. <u>Exploring Rational Exponents</u>

A rational exponent represents two separate operations attached to the base number. Each number of the fraction represents a different operation. The "top" number of the fraction represents the exponent that is applied to the base number, the "bottom" number represents the root that is applied.

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ or } \left(\sqrt[n]{a}\right)^m$$

C. <u>Examples</u>

  1) Determine the exact value of the following.

a) $\left(\dfrac{9}{4}\right)^{\frac{1}{2}} = \dfrac{(9)^{\frac{1}{2}}}{(4)^{\frac{1}{2}}}$

$= \dfrac{\sqrt{9}}{\sqrt{4}} = \boxed{\dfrac{3}{2}}$

b) $4^{-\frac{3}{2}} = \dfrac{1}{4^{\frac{3}{2}}}$

$= \dfrac{1}{\sqrt{4^3}} = \boxed{\dfrac{1}{8}}$

c) $4^{\frac{5}{2}} = \sqrt{4^5} \text{ or } \left(\sqrt{4}\right)^5$

$= \boxed{32}$

2) Simplify the following. Write your answer as a **power** and a **radical**.

a) $y^{\frac{5}{7}} \div y^{\frac{2}{7}} = y^{\frac{5}{7} - \frac{2}{7}} = y$

$= \boxed{y^{\frac{3}{7}}}$ or $\boxed{\sqrt[7]{y^3}}$

b) $\left(\dfrac{x^2}{y}\right)^{-\frac{1}{2}} = \dfrac{x^{2 \cdot -\frac{1}{2}}}{y^{-\frac{1}{2}}} = \dfrac{x^{-1}}{y^{-\frac{1}{2}}}$

$= \boxed{\dfrac{y^{\frac{1}{2}}}{x}}$ or $\boxed{\dfrac{\sqrt{y}}{x}}$

c) $\left(4m^{\frac{3}{4}}\right)\left(3m^{-\frac{1}{2}}\right) = 12m^{\frac{3}{4} + -\frac{1}{2}}$

$= \boxed{12m^{\frac{1}{4}}}$ or $\boxed{12\sqrt[4]{m}}$

d) $\sqrt{\sqrt{x^3}} = \left(x^{\frac{3}{2}}\right)^{\frac{1}{2}} = x^{\frac{3}{2} \cdot \frac{1}{2}}$

$= \boxed{x^{\frac{3}{4}}}$ or $\boxed{\sqrt[4]{x^3}}$

Assignment: Rational Exponents Assignment #1 – 8

Class Ex. #6 Write an equivalent expression using exponents.

a) $\sqrt{\sqrt{\sqrt{a^3}}}$

b) $\sqrt{\sqrt[3]{64x^6}}$

c) $\dfrac{1}{\sqrt[3]{\sqrt{\sqrt{5y^{12}}}}}$

Complete Assignment Questions #1 - #18

Assignment

1. Write each power as a radical.

a) $a^{\frac{4}{3}}$

b) $b^{\frac{3}{2}}$

c) $c^{\frac{1}{4}}$

d) $x^{-\frac{2}{3}}$

e) $y^{-\frac{1}{3}}$

f) $5h^{\frac{3}{5}}$

g) $(5h)^{\frac{2}{5}}$

h) $-r^{\frac{5}{4}}$

i) $(-r)^{\frac{5}{4}}$

j) $2x^{-\frac{1}{2}}$

2. Write each radical as a power.

a) $\sqrt[5]{x^3}$

b) $\sqrt[5]{a^4}$

c) $\sqrt{d^5}$

d) $\dfrac{1}{\sqrt[4]{y}}$

e) $\dfrac{1}{\sqrt[4]{h^5}}$

f) $\left(\sqrt[4]{2y-3}\right)^{-3}$

g) $-\sqrt[3]{x^2}$

h) $\sqrt[3]{-x^2}$

i) $\sqrt[3]{(-x)^2}$

j) $\sqrt[3]{(-x)^{-2}}$

3. Simplify the following. Write each expression as a power and as a radical.

a) $x^{\frac{7}{2}} \times x$

b) $y^{\frac{6}{5}} \div y^{\frac{4}{5}}$

c) $(a^{\frac{2}{3}})^{\frac{3}{4}}$

d) $(e^3 f)^{\frac{3}{2}}$

e) $x^{\frac{1}{2}} \times x^{-1}$

f) $y^{\frac{2}{7}} \div y^{\frac{5}{7}}$

g) $\left(\dfrac{x}{y^4}\right)^{\frac{1}{2}}$

h) $\left(\dfrac{x^2}{y}\right)^{-\frac{3}{2}}$

4. Write as a power and evaluate.

a) $\sqrt{\sqrt[3]{64}}$

b) $\dfrac{1}{\sqrt[4]{625}}$

c) $\sqrt{\sqrt{2401}}$

5. Simplify the following. Write each expression as a power and as a radical.

a) $2x^{\frac{3}{8}} \times 5x^{-\frac{1}{8}}$

b) $64(a^{\frac{1}{2}})^{\frac{1}{3}}$

c) $((64a)^{\frac{1}{3}})^{\frac{1}{2}}$

d) $(64a^{\frac{1}{3}})^{\frac{1}{2}}$

e) $\dfrac{y^{\frac{2}{3}} y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

f) $\dfrac{a^3 b^{\frac{1}{2}}}{b^3 (a^{\frac{3}{2}})^2}$

g) $\dfrac{10x^{\frac{1}{3}}}{5x^{-\frac{3}{3}}}$

h) $\dfrac{(a^4)^{\frac{1}{3}}}{9} + \dfrac{a}{81^{\frac{3}{4}}}$

6. Simplify. Write each expression in simplest radical form and as a power.

a) $\sqrt[3]{27x^7}$

b) $\sqrt[4]{81a^3}$

c) $\sqrt[3]{(-270y)}$

d) $\left(\sqrt[4]{x^3}\right)\left(\sqrt{x}\right)$

e) $\sqrt[3]{a} \times \sqrt[3]{a}$

f) $\left(\sqrt[4]{x^4y^3}\right)^{\frac{3}{2}}$

7. Write an equivalent expression using exponents.

a) $\sqrt{\sqrt{x^5}}$

b) $\sqrt[3]{\sqrt{a^8}}$

c) $\sqrt[3]{\sqrt{16y^{12}}}$

d) $\sqrt[3]{\sqrt[4]{x^{\frac{2}{3}}}}$

e) $\left(\dfrac{25\sqrt[3]{x^5}}{5x^{\frac{1}{3}}}\right)^2$

f) $\left(\sqrt[4]{\sqrt[5]{y^{\frac{1}{3}}}}\right)^5$

8. Simplify and express each as a power with positive exponents.

a) $\dfrac{6x^{-\frac{1}{4}} \cdot 2x^{\frac{5}{2}}}{-3x^{-\frac{3}{4}}}$

b) $\left(\dfrac{768b^{-1}}{3c^{-1}}\right)^{-\frac{3}{4}}$

c) $\dfrac{a^{\frac{1}{3}}(a^{\frac{2}{3}})^{-\frac{5}{3}}}{a^{\frac{4}{3}}}$

Answer Key

Unless otherwise indicated in the question, radicals can be given in the form $\sqrt[n]{x^m}$ or $\left(\sqrt[n]{x}\right)^m$ and power can be given in the form x^{-n} or $\dfrac{1}{x^n}$. Equivalent versions of some answers are possible.

1. a) $\sqrt[5]{a^4}$ b) $\sqrt{b^3}$ c) $\sqrt[6]{c}$ d) $\dfrac{1}{\sqrt[5]{x^2}}$ e) $\dfrac{1}{\sqrt[3]{y}}$ f) $5\sqrt[3]{h^2}$

   g) $\sqrt[3]{(5h)^2}$ h) $-\sqrt[4]{r^5}$ i) $\sqrt[4]{(-r)^5}$ j) $\dfrac{2}{\sqrt{x}}$

2. a) $x^{\frac{3}{5}}$ b) $a^{\frac{4}{5}}$ c) $d^{\frac{5}{2}}$ d) $y^{-\frac{1}{4}}$ e) $h^{-\frac{5}{4}}$ f) $(2y-3)^{-\frac{3}{4}}$ g) $-x^{\frac{2}{3}}$ h) $-x^{\frac{2}{3}}$ i) $(-x)^{\frac{2}{3}}$

   j) $(-x)^{-\frac{2}{3}}$

3. a) $x^{\frac{9}{2}} = \sqrt{x^9}$ b) $y^{\frac{2}{5}} = \sqrt[5]{y^2}$ c) $a^{\frac{3}{10}} = \sqrt[10]{a^3}$ d) $e^{\frac{9}{2}}f^{\frac{3}{2}} = e^4 f\sqrt{ef}$

   e) $\dfrac{1}{x^{\frac{1}{2}}} = \dfrac{1}{\sqrt{x}}$ f) $\dfrac{1}{y^{\frac{3}{7}}} = \dfrac{1}{\sqrt[7]{y^3}}$ g) $\dfrac{x^{\frac{1}{2}}}{y^2} = \dfrac{\sqrt{x}}{y^2}$ h) $\dfrac{y^{\frac{3}{2}}}{x^3} = \dfrac{\sqrt{y^3}}{x^3}$

4. a) $64^{\frac{1}{6}} = 2$ b) $625^{-\frac{1}{4}} = \dfrac{1}{5}$ c) $2401^{\frac{1}{4}} = 7$

5. a) $10x^{\frac{1}{4}} = 10\sqrt[4]{x}$ b) $64a^{\frac{1}{6}} = 64\sqrt[6]{a}$ c) $2a^{\frac{1}{6}} = 2\sqrt[6]{a}$ d) $8a^{\frac{1}{6}} = 8\sqrt[6]{a}$

   e) $y^{\frac{11}{12}} = \sqrt[12]{y^{11}}$ f) $\dfrac{1}{b^{\frac{5}{2}}} = \dfrac{1}{\sqrt{b^5}}$ g) $2x^{\frac{4}{5}} = 2\sqrt[5]{x^4}$ h) $3a^{\frac{1}{3}} = 3\sqrt[3]{a}$

6. a) $3x^2\sqrt[3]{x} = 3x^{\frac{7}{3}}$ b) $3\sqrt[4]{a^3} = 3a^{\frac{3}{4}}$ c) $-3\sqrt[3]{10y} = -3(10y)^{\frac{1}{3}}$

   d) $\sqrt[4]{x^5} = x^{\frac{5}{4}}$ e) $\sqrt[3]{a^2} = a^{\frac{2}{3}}$ f) $\sqrt[8]{x^{12}y^9} = x^{\frac{3}{2}}y^{\frac{9}{8}}$

7. a) $x^{\frac{5}{4}}$ b) $a^{\frac{4}{3}}$ c) $4^{\frac{1}{3}}y^2$ d) $x^{\frac{1}{18}}$ e) $25x^{\frac{8}{3}}$ f) $y^{\frac{1}{12}}$

8. a) $-4x^3$ b) $\dfrac{b^{\frac{3}{8}}}{8c^{\frac{3}{8}}}$ c) $\dfrac{1}{a^{\frac{5}{3}}}$