# Sine Law

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9:52 AM

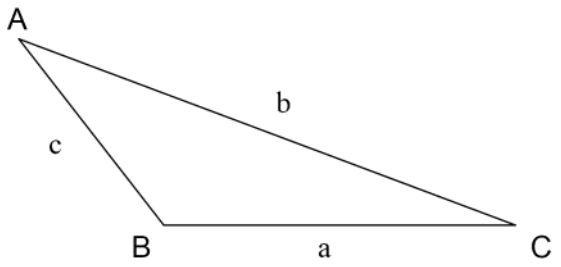
PRE-CALCULUS 11
TRIGONOMETRY
SINE LAW

So far we've been able to work with right-angle triangles when using trigonometric functions. Although these are really useful triangles, there are lots of other types of triangles besides right angles. We need a way to deal with those.

If we're working with a non-right angle triangle, that means there's no hypotenuse, so we can't use Pythagorean Theorem. Instead, we have 2 tools called the ***Sine Law*** and the ***Cosine Law***. Let's take a look at the Sine Law first.

A. Sine Law

Look at the following triangle:




$$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$$

**Important Points About Sine Law**

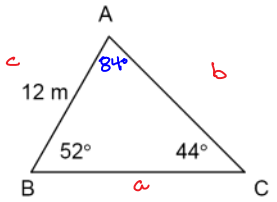
1) The triangle needs to be labelled the way it is in the picture above: side "***a***" is opposite angle "***A***", side "***b***" is opposite angle "***B***", and side "***c***" is opposite angle "***C***" . ***Remember that in a non-right triangle "c" is not the hypotenuse)***

2) In order to use Sine Law you must have one complete ratio (angle + side) and one incomplete ratio.

3) Remember to check for an Ambiguous Case that may give two answers.

B. Examples

1) Calculate the length of BC. Round your answer to one decimal.




$\frac{\sin C}{c} = \frac{\sin A}{a}$

$12a\left[\frac{\sin 44^\circ}{12} = \frac{\sin 84^\circ}{a}\right]$

$\frac{\sin 44^\circ}{\sin 44^\circ} a = \frac{(12)(\sin 84^\circ)}{\sin 44^\circ}$

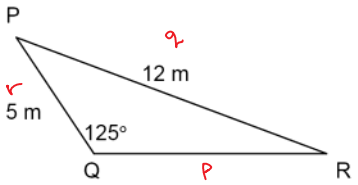
$a = 17.18004\ldots$

$BC = 17.2\text{ m}$

$180^\circ - 52^\circ - 44^\circ$

$\angle A = 84^\circ$

2) Calculate $\angle R$ to the nearest degree.




$\frac{\sin Q}{q} = \frac{\sin R}{r}$

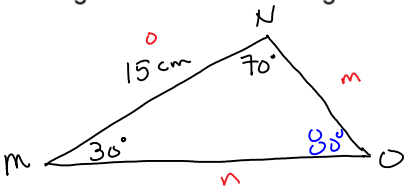
$60\left[\frac{\sin 125^\circ}{12} = \frac{\sin R}{5}\right]$

$\frac{(5)(\sin 125^\circ)}{12} = \frac{12 \sin R}{12}$

$\sin R = 0.34131\ldots$

$\angle R = 20^\circ$

3) In $\Delta MNO$, $\angle M = 30^0$, $\angle N = 70^0$, and MN = 15 cm. Solve for all missing parts. Round all angles to the nearest degree and all lengths to one decimal.




$\frac{\sin O}{o} = \frac{\sin M}{m}$

$15m\left[\frac{\sin 80^\circ}{15} = \frac{\sin 30^\circ}{m}\right]$

$\frac{\sin 80^\circ}{\sin 80^\circ} m = \frac{(15)(\sin 30^\circ)}{\sin 80^\circ}$

$ON = 7.6\text{ cm}$

$\frac{\sin O}{o} = \frac{\sin N}{n}$

$15n\left[\frac{\sin 80^\circ}{15} = \frac{\sin 70^\circ}{n}\right]$

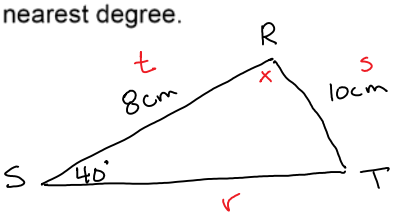
$\frac{\sin 80^\circ}{\sin 80^\circ} n = \frac{(15)(\sin 70^\circ)}{\sin 80^\circ}$

$MO = 14.3\text{ cm}$

$180^\circ - 30^\circ - 70^\circ$

$\angle O = 80^\circ$

4) ) In $\Delta RST$, RS = 8 cm, RT = 10 cm and $\angle RST = 40^0$. Find the measure of $\angle SRT$ to the nearest degree.



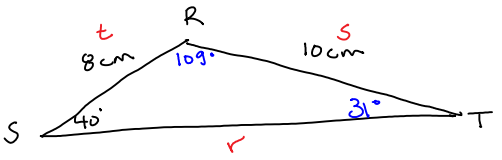

One Triangle.

$\frac{\sin S}{S} = \frac{\sin T}{t}$

$40\left[\frac{\sin 40^\circ}{10} = \frac{\sin T}{8}\right]$

$\frac{(4)(\sin 40^\circ)}{5} = \frac{5 \sin T}{5}$

$\sin T = 0.51423...$

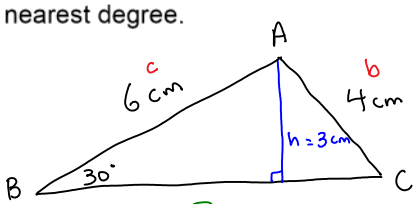



$\angle T = 31^\circ$

$180^\circ - 40^\circ - 31^\circ$

$\boxed{\angle SRT = 109^\circ}$

5) In $\Delta ABC$, AB = 6 cm, AC = 4cm and $\angle ABC = 30^0$. Find the measure of $\angle ACB$ to the nearest degree.

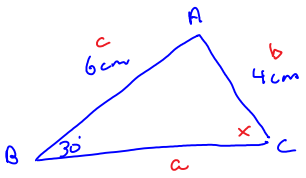



$6\left[\sin 30^\circ = \frac{h}{6}\right]$

$(6)(\sin 30^\circ) = h$

$h = 3\text{cm}$

2 triangles




$\frac{\sin B}{b} = \frac{\sin c}{c}$

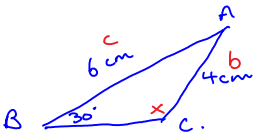
$12\left[\frac{\sin 30^\circ}{4} = \frac{\sin C}{6}\right]$

$\frac{(3)(\sin 30^\circ)}{2} = \frac{2 \sin C}{2}$

$\sin C = 0.75$

$\boxed{\angle ACB = 49^\circ}$

acute.
Q1




reference angle = 49°

$180^\circ - 49^\circ$

$\boxed{\angle ACB = 131^\circ}$

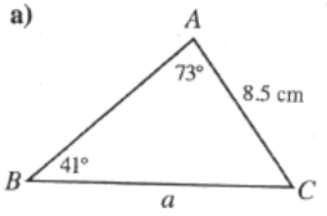
obtuse.
Q2

Assignment: Sine Law Assignment #1, 2, 3, 4, 5, 6, 10, 11, 12, 13

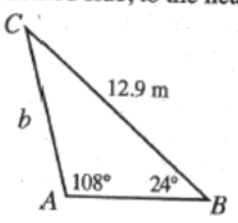
# Assignment

**1.** In each case find the length of the indicated side, to the nearest tenth.

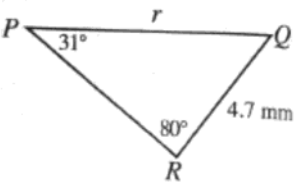
**a)**




**b)**

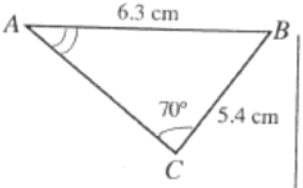



**c)**

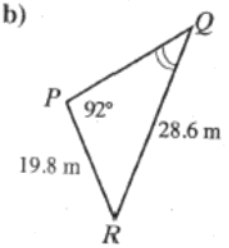



**2.** In each case find the measure of the indicated angle, to the nearest degree.

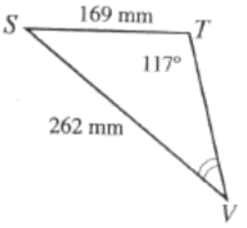
**a)**




**b)**



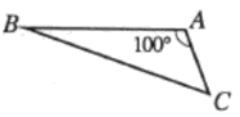

**c)**



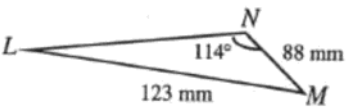
**3.** In $\Delta ABC$, angle $A = 49°$, angle $B = 57°$ and $a = 8$. Calculate $b$, to the nearest tenth.

**4.** In $\Delta ABC$, angle $A = 53°$, angle $B = 61°$ and $b = 2.8$. Calculate $a$, to the nearest tenth.

**5.** In $\Delta ABC$, angle $A = 100°$, $a = 7.9$ and $b = 4.5$. Calculate angle $B$, to the nearest degree.




**6.** In $\Delta LMN$, angle $LNM = 114°$, $LM = 123$ mm and $MN = 88$ mm. Calculate $\angle LMN$, to the nearest degree.



Multiple Choice

**10.** In triangle $PQR$, angle $P = 20°$, angle $R = 150°$ and $QR = 6$ m. The length in m of $PQ$ is

**A.** 4.1
**B.** 8.8
**C.** 15.2
**D.** 17.3

**11.** Triangle $LMN$ is obtuse angled at $M$ and $\angle MLN = 40°$. Sin $LNM$ is equal to

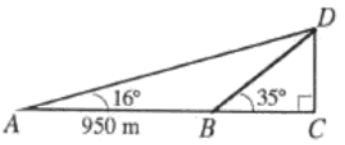
**A.** $\dfrac{MN}{LM \sin 40°}$
**B.** $\dfrac{LM}{MN \sin 40°}$
**C.** $\dfrac{LM \sin 40°}{MN}$
**D.** $\dfrac{MN \sin 40°}{LM}$

**12.** In $\Delta ABC$, $\angle A = 30°$, $BC = 10$ units and $AC = 15$ units. If $\angle B$ is acute-angled, then $\angle C$ is

**A.** 19.4°
**B.** 48.6°
**C.** 101.4°
**D.** 130.6°

Numerical Response

**13.** From a point $A$, level with the foot of a hill, the angle of elevation of the top of the hill is 16°. From a point $B$, 950 metres nearer the foot of the hill, the angle of elevation of the top is 35°.
The height of the hill, $DC$, to the nearest metre, is ____ .




(Record your answer in the numerical response box from left to right)

***Answer Key***

**1. a)** 12.4 cm **b)** 5.5 m **c)** 9.0 mm **2. a)** 54° **b)** 44° **c)** 35°

**3.** 8.9 **4.** 2.6 **5.** 34° **6.** 25° **7. a)** 49° **b)** 138 m

**8.** 39 m **10.** B **11.** C **12.** C **13.**

| | | | |
|---|---|---|---|
| 4 | 6 | 1 | |

**14. a)** 34° or 146° **b)** 43° **15.** 61° or 119° **16.** D