Solving Rational Equations

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A. Definitions

1) **rational equation:** an mathematical equation that contains rational expressions.

$$\frac{2x}{3} + \frac{1}{5} = 4x \quad , \quad \frac{1}{x+2} + \frac{3}{x} = 4x$$

2) **extraneous root:** a solution to a rational equation that when checked does not satisfy the original equation.

B. Examples

a) $4x\left[\dfrac{5}{2x} + \dfrac{3}{4} = \dfrac{9}{4x}\right]$ $\quad x \neq 0$

$$10 + 3x = 9$$
$$-10 \qquad -10$$

$$\frac{3x}{3} = \frac{-1}{3}$$

$$x = -\frac{1}{3}$$

$$\boxed{x = -\frac{1}{3}}$$

b) $x\left[4 + \dfrac{2}{x} = 7 + \dfrac{3}{x}\right]$ $\quad x \neq 0$

$$4x + 2 = 7x + 3$$
$$-7x \;-2 \qquad -7x \;-2$$

$$\frac{-3x}{-3} = \frac{1}{-3}$$

$$x = -\frac{1}{3}$$

$$\boxed{x = -\frac{1}{3}}$$

c) $\dfrac{2}{x+2} \ominus \dfrac{x^2+4}{(x+2)(x-2)} = \dfrac{-x}{x-2}$ $x \neq -2, 2$

$(x+2)(x-2)\left[\dfrac{2}{x+2} + \dfrac{-x^2-4}{(x+2)(x-2)} = \dfrac{-x}{x-2}\right]$

$2(x-2) - x^2 - 4 = -x(x+2)$

$2x - 4 - x^2 - 4 = -x^2 - 2x$

$2x - 4 - 4 = -2x$

$2x - 8 = -2x$
$-2x \qquad\quad -2x$

$\dfrac{-8}{-4} = \dfrac{-4x}{-4}$

$2 = x$
Extraneous Root

$\boxed{\text{No Solution}}$

Assignment: Rational Equations Assignment #6, 7, 8, 9

5. Solve and verify.

a) $\dfrac{y}{2} - \dfrac{y}{5} = 18$

b) $2a - \dfrac{a+2}{3} = \dfrac{a+3}{4}$

c) $\dfrac{1}{5}(3x+1) - 7 = \dfrac{1}{2}(x-1) - 2x$

6. Solve and verify

a) $\dfrac{6a+3}{2a-3} = \dfrac{3}{2}$

b) $\dfrac{2}{m+1} = \dfrac{8m}{m+1} - 3$

c) $\dfrac{5a-3}{a+7} = \dfrac{5a-14}{a+1}$

7. Solve

a) $\dfrac{2x+1}{x-3} - \dfrac{4x-1}{2x-3} = 0$

b) $\dfrac{6y-2}{3y-2} - \dfrac{2y+6}{y+6} = 0$

c) $\dfrac{4a+9}{2a} - \dfrac{3}{4} = 2$

d) $\dfrac{5}{3x-1} + \dfrac{3x}{3x+1} = 1$

e) $\dfrac{8x}{2x+3} - \dfrac{x+3}{x+7} = 3$
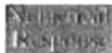
f) $\dfrac{4x+3}{2x-1} - 2 = \dfrac{6x+2}{2x-1}$

Multiple Choice 8. If $\dfrac{2}{4-y} = 3$, then y equals

    A. $\dfrac{5}{2}$

    B. $\dfrac{10}{3}$

    C. -2

    D. $\dfrac{14}{3}$

9. The solution to the equation $\dfrac{7}{a+6} - \dfrac{3}{a} = \dfrac{4}{a+6}$ is

    A. $a = 18$

    B. $a = -6$

    C. $a = 0$

    D. there is no solution.

Numerical Response 10. The solution to the rational equation $\dfrac{4}{x} + \dfrac{2x}{x-4} - 2 = 0$, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) 6 b) $\dfrac{3}{2}$ c) 9   2. a) $\dfrac{2}{7}$ b) $\dfrac{11}{5}$ c) 49   3. a) $\dfrac{7}{4}$ b) 0

4. a) $\dfrac{5}{3}$ b) $\dfrac{26}{11}$ 5. a) 60 b) 1 c) 3   6. a) $-\dfrac{5}{2}$ b) 1 c) 5

7. a) $\dfrac{2}{3}$ b) 0 c) 6 d) $-\dfrac{1}{2}$ e) -18 f) no solution $\left(\dfrac{1}{2}\text{ is a nonpermissible value}\right)$

8. B   9. D   10.

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