

The Factored Form of a Quadratic Function

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PRE-CALCULUS 11
QUADRATIC FUNCTIONS
THE FACTORED FORM OF A QUADRATIC FUNCTION

A. Definitions

1. **general form:** any quadratic function that can be written in the form:

$$y = ax^2 + bx + c \text{ or } f(x) = ax^2 + bx + c.$$

2. **standard form:** any quadratic function that can be written in the form:

$$y = a(x - p)^2 + q \text{ or } f(x) = a(x - p)^2 + q$$

3. **factored form:** any quadratic function that can be written in the form:

$$y = a(x - r_1)(x - r_2) \text{ or } f(x) = a(x - r_1)(x - r_2)$$

↑ roots!

B. The Factored Form of a Quadratic Function

The factored form of the quadratic function $y = a(x - r_1)(x - r_2)$, easily allows us to find the x-intercepts of the function as well as the expansion/compression value and whether the function is upright or inverted.

- 1) Change the following function into the factored form and then describe the characteristics of the graph.

a) $y = 2x^2 + 12x + 10$

$$\begin{array}{r} 5 \\ \textcircled{5} \times \textcircled{1} \\ 6 \end{array}$$

$$y = 2(x^2 + 6x + 5)$$

$$y = 2(x + 5)(x + 1)$$

- x-int $(-5, 0)$ & $(-1, 0)$
- upright & vertically expanded.

b) $y = -\frac{1}{3}\left(x + \frac{3}{2}\right)^2 + \frac{27}{4}$

$$\begin{array}{r} -18 \\ 6 \times -3 \\ 3 \end{array}$$

$$y = -\frac{1}{3}\left(x + \frac{3}{2}\right)\left(x + \frac{3}{2}\right) + \frac{27}{4}$$

$$y = -\frac{1}{3}\left(x^2 + 3x + \frac{9}{4}\right) + \frac{27}{4}$$

$$y = -\frac{1}{3}x^2 - x - \frac{3}{4} + \frac{27}{4}$$

$$y = -\frac{1}{3}x^2 - x + 6$$

$$y = -\frac{1}{3}(x^2 + 3x - 18)$$

$$y = -\frac{1}{3}(x + 6)(x - 3)$$

- x-int $(-6, 0)$ & $(3, 0)$
- inverted & vertically compressed.

- 2) Write the factored form equation $y = a(x - r_1)(x - r_2)$ of a quadratic function that has zeros of -3 and 5 and passes through the point $A(2, -10)$.

roots

$$y = a(x - r_1)(x - r_2)$$

$$y = a(x + 3)(x - 5)$$

$$(-10) = a(2 + 3)(2 - 5)$$

$$-10 = a(5)(-3)$$

$$\frac{-10}{-15} = \frac{-15a}{-15}$$

$$\underline{a = \frac{2}{3}}$$

$$y = \frac{2}{3}(x + 3)(x - 5)$$

- 3) Write the general form equation $y = ax^2 + bx + c$ of a quadratic function that has zeros of 1 and 7 and passes through the point $B(3, 16)$.

roots

$$y = a(x - r_1)(x - r_2)$$

$$y = a(x - 1)(x - 7)$$

$$(16) = a(3 - 1)(3 - 7)$$

$$16 = a(2)(-4)$$

$$\frac{16}{-8} = \frac{-8a}{-8}$$

$$\underline{a = -2}$$

$$y = -2(x - 1)(x - 7)$$

$$y = -2(x^2 - 8x + 7)$$

$$y = -2x^2 + 16x - 14$$

- 4) Write the factored form equation $y = a(x - p)(x - r_2)$ of a quadratic function that has a vertex of $(1, -18)$ and an x-intercept of $(4, 0)$.

$$y = a(x - p)^2 + q$$

$$y = a(x - 1)^2 - 18$$

$$(0) = a(4 - 1)^2 - 18$$

$$0 = 9a - 18$$

$$\frac{18}{9} = \frac{9a}{9}$$

$$\underline{a = 2}$$

$$\begin{array}{r} -8 \\ -4 \times 2 \\ -2 \end{array}$$

$$y = 2(x - 1)^2 - 18$$

$$y = 2(x - 1)(x - 1) - 18$$

$$y = 2(x^2 - 2x + 1) - 18$$

$$y = 2x^2 - 4x + 2 - 18$$

$$y = 2x^2 - 4x - 16$$

$$y = 2(x^2 - 2x - 8)$$

$$y = 2(x - 4)(x + 2)$$

Assignment:

Pg. 306 #3, 4, 5, 8, 9, 10, 12