

Transforming the Graph of a Quadratic Function

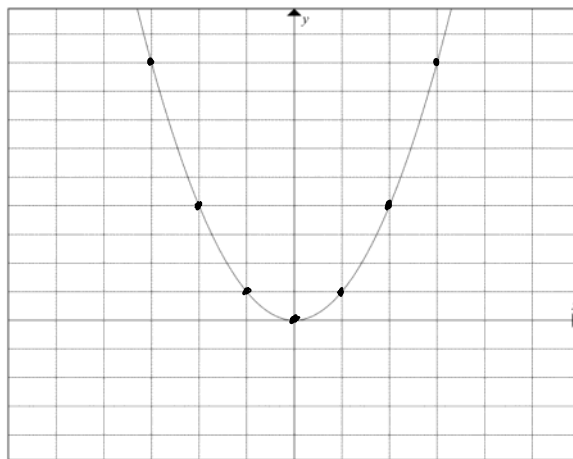
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PRE-CALCULUS 11 QUADRATIC FUNCTIONS TRANSFORMING THE GRAPH OF A QUADRATIC FUNCTION

A. The Different Properties of a Quadratic Function

The simplest quadratic function of all is the function $y = x^2$ and is referred to as **The Parent Graph**. All other transposed parabolas are based on this simplest one.

$$y = x^2$$



Vertex (0, 0)

Over 1, Up 1
Over 2, Up 4
Rule

Left or Right	Up
1 Right	1 Up.
1 Left	1 Up
2 Right	4 Up.
2 Left	4 Up.
3 Right	9 Up.
3 Left.	9 Up.

Comparing the function $y = x^2$ to the function $y = (x - p)^2$.

Function	Value of p	Opening Up/Down	Vertex	Axis of Symmetry	Congruent to $y = x^2$?
$y = x^2$	0	Up	(0,0)	$x = 0$	Yes
$y = (x - 4)^2$	4	Up.	(4,0)	$x = 4$	Yes
$y = (x + 4)^2$	-4	Up.	(-4,0)	$x = -4$	Yes.
$y = (x - 7)^2$	7	Up.	(7,0)	$x = 7$	Yes
$y = (x + 7)^2$	-7	Up.	(-7,0)	$x = -7$	Yes.

What does the "p" value do to the vertex of the function?

The "p" value shifts the vertex left or right

Graph the following functions:

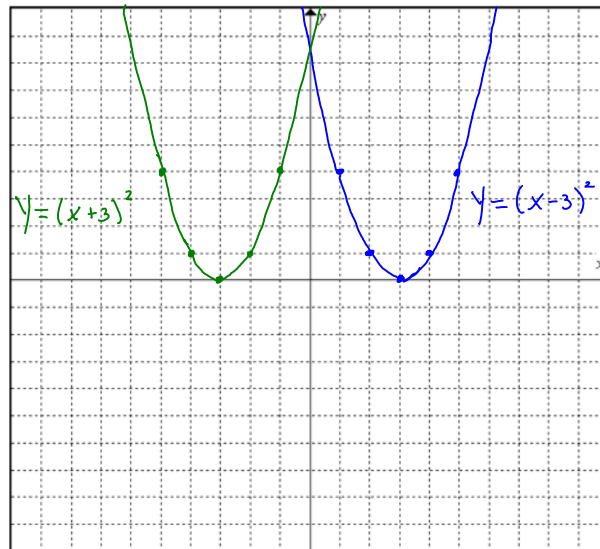
$$y = (x - 3)^2 \text{ and } y = (x + 3)^2$$

$$y = (x - 3)^2$$

$p = 3$
vertex (3,0)
Over 1, Up 1
Over 2, Up 4

$$y = (x + 3)^2$$

$p = -3$
vertex (-3,0)
Over 1, Up 1
Over 2, Up 4



Comparing the function $y = x^2$ to the function $y = x^2 + q$.

Function	Value of q	Opening Up/Down	Vertex	Axis of Symmetry	Congruent to $y = x^2$?
$y = x^2$	0	Up	(0,0)	$x = 0$	Yes
$y = x^2 + 4$	4	Up.	(0,4)	$x = 0$	Yes.
$y = x^2 - 4$	-4	Up	(0,-4)	$x = 0$	Yes
$y = x^2 + 7$	7	Up	(0,7)	$x = 0$	Yes.
$y = x^2 - 7$	-7	Up	(0,-7)	$x = 0$	Yes

What does the "q" value do to the vertex of the function?

The "q" value shifts the vertex up or down.

Graph the following functions:

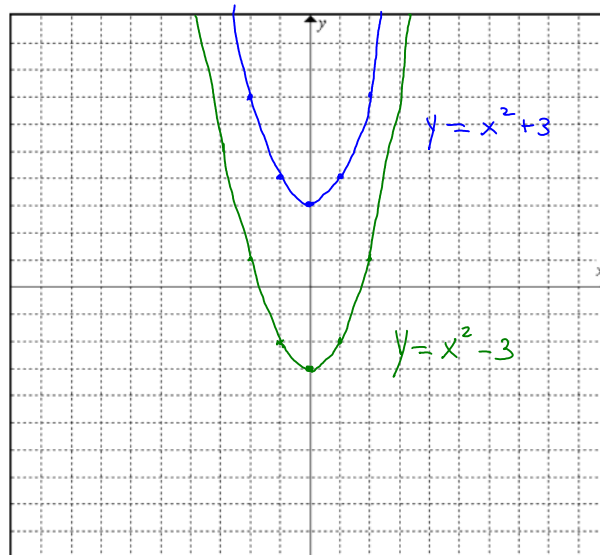
$$y = x^2 + 3 \text{ and } y = x^2 - 3$$

$$y = x^2 + 3$$

$q = 3$
vertex (0,3).
Over 1, Up 1
Over 2, Up 4

$$y = x^2 - 3$$

$q = -3$
vertex (0,-3)
Over 1, Up 1
Over 2, Up 4



Comparing the function $y = x^2$ to the function $y = ax^2$.

Function	Value of a	Opening Up/Down	Vertex	Axis of Symmetry	Congruent to $y = x^2$?
$y = x^2$	1	Up.	(0,0)	$x = 0$	Yes
$y = -x^2$	-1	Down	(0,0)	$x = 0$	Yes but inverted
$y = 3x^2$	3	Up	(0,0)	$x = 0$	No it is vertically expanded.
$y = -3x^2$	-3	Down	(0,0)	$x = 0$	No it is inverted and vertically expanded.
$y = \frac{1}{3}x^2$	$\frac{1}{3}$	Up	(0,0)	$x = 0$	No it is vertically compressed.

What does the "a" value do to the graph of the function?

The "a" value causes the graph to be vertically compressed, vertically expanded, or inverted.

$a > 1$ vertical expansion
 $0 < a < 1$ vertical compression
 $a < 0$ inverted.

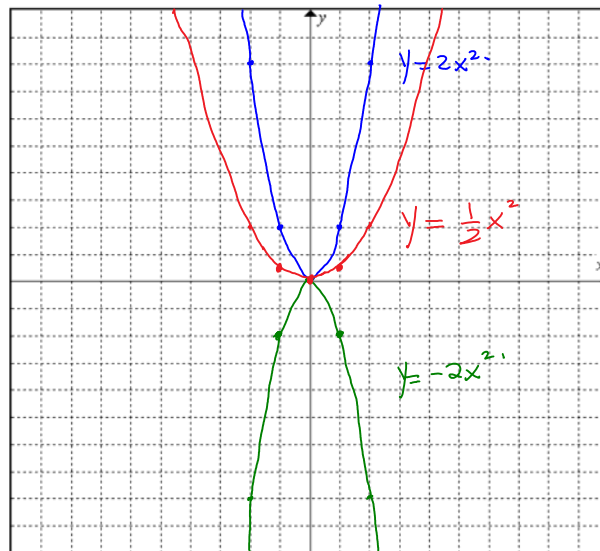
Graph the following functions:

$$y = 2x^2, \quad y = -2x^2 \quad \text{and} \quad y = \frac{1}{2}x^2$$

$y = 2x^2$
 $a = 2$
 vertex (0,0)
 Over 1, Up $1(2) = 2$
 Over 2, Up $4(2) = 8$

$y = -2x^2$
 $a = -2$
 vertex.
 Over 1, Up $1(-2) = -2$
 Over 2, Up $4(-2) = -8$

$y = \frac{1}{2}x^2$
 $a = \frac{1}{2}$
 Over 1, Up $1(\frac{1}{2}) = \frac{1}{2}$
 Over 2, Up $4(\frac{1}{2}) = 2$.



B. Translating Functions

The graph of $y = x^2$ is translated as below. Without graphing, write the equation of the graph in its new position.

- 1) a translation of 10 units down.

$$q = -10$$
$$y = x^2 - 10$$

- 2) a translation of 4 units to the right.

$$p = 4$$
$$y = (x - 4)^2$$

- 3) a vertical compression of $\frac{1}{5}$.

$$a = \frac{1}{5}$$
$$y = \frac{1}{5}x^2$$

- 4) an vertical expansion of 6, ^{inverted} and reflected in the x-axis.

$$a = -6$$
$$y = -6x^2$$